MAKING SENSE OF MATHEMATICS

1. The definition of logarithm is if $a^{x}=y$, then $\log _{a} y=x$, and if $\log _{a} y=x$, then $a^{x}=y$.
a. Complete the tables for an exponential function base 10 and a logarithmic function base 10.

| $x$ | $10^{x}$ |
| :---: | :---: |
| 0 | $10^{0}=1$ |
| 1 |  |
| 2 |  |
| 3 | $10^{3}=1000$ |
| 4 |  |
| 5 |  |
| 6 | $10^{6}=1000000$ |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |


| $y$ | $\log _{10} y$ |
| :---: | :---: |
| 1 | 0 |
| 10 |  |
| 100 |  |
| 1000 |  |
| 10000 |  |
| $10^{5}$ |  |
| $10^{6}$ |  |
| $10^{7}$ |  |
| $10^{8}$ |  |
| $10^{9}$ |  |
| $10^{10}$ |  |

b. Ten raised to what power is $1,000,000$ ?
c. How can the definition of logarithms help you find $\log _{10} 1000000$ ?
d. Using the table, estimate $\log _{10} 99,932$ to the nearest whole number.
e. Using the table, estimate $10^{3.1}$.
2. Complete the tables below. The base is three in both tables.

| $x$ | $3^{x}$ |
| :---: | :---: |
| 0 | 1 |
| 1 | 3 |
| 2 |  |
| 3 | 27 |
| 4 |  |
| 5 |  |


| $y$ | $\log _{3} y$ |
| :---: | :---: |
| 1 | 0 |
| 3 | 1 |
| 9 |  |
| 27 | 3 |
| 81 |  |
| 243 |  |

a. Without using a calculator, compute the following base three logarithms.
i. $\quad \log _{3}(81)$
ii. $\quad \log _{3}(243)$
iii. $\log _{3}(1)$
iv. $\log _{3}\left(\frac{1}{3}\right)$
v. $\log _{3}\left(\frac{1}{9}\right)$
3. Moore's Law states, informally, that the computing power of a chip doubles every two years.
a. Make a table showing how the computing power of a chip increases, where $n$ is the number of doubling periods.

| n | $2^{\mathrm{n}}$ |
| :--- | :--- |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

b. According to Moore's Law, how long will it take the computing power of a chip to increase by a factor of 64?
c. According to Moore's Law, by what factor will the computing power of the chip increase in 16 years?

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4. Assume the population (p) of a virus in a human body triples every hour.
a. If we start with 1 virus in a body, how many will there be in three hours?

| $t$ | $3^{t}$ |
| :--- | :--- |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

b. How long will it take for the population of viruses to be 243 ?
c. How many viruses will there be in one day?
d. Is the equation below a valid representation for the number of viruses in a human body? Why or why not?

$$
t=\log _{3}(p)(t=\text { time in hours, } p=\text { population })
$$

5. The following is a graph of $y=4^{x}$. Use the graph to estimate $\log _{4}(8000)$.

