Making Sense of Log Properties

Video Script

Scene		Full Transcript
1	**Nate:**	Hey you need a wrench or anything? Ok, I'll just stand over here. Hey it's Nate! I'm no mechanic; I barely know how to check the air in my tires. But today I'm with air mechanics of the National Guard 248 aviation support battalion where I'm learning how they keep their unit moving. They work on everything. From Humvees to this tactical vehicle. These guys are so good, they can do most of this stuff in their sleep. But every now and then they need to consult a manual to get the job done. In a sense, math is like that. Take log properties. We use them all the time, but to really understand why they work, we need to prove them. To do this, we're going to consult our math manual. Lets look under the hood and get another problem solved.
2	**Voice-Over Nate:**	We'll start by looking at this property. Log the quantity x times y equals the log of x plus the log of y. If we want to prove this, we need to show that the left hand side is equivalent to the right hand side.
	Nate:	Figuring out where to begin is sometimes the hardest part. We can think about what we already know, kind of like browsing through our math manual.
	Voice-Over Nate:	Well we know the definition of logarithms, and since a logarithm is also an exponent, lets also keep in the mind the Properties of Exponents. We need something with log x and log y. We can try letting n equal log x and m equal log y. So, using the definition of logarithms, if n equals log x then x equals 10 to the n. And this is similarly true for y equals 10 to the m. Since we're working with the log of the quantity x, y, we should multiply x and y. X times y equals 10 to the n times 10 to the m. Okay, so what's next? Using the property of exponents for multiplication, x times y equals 10 to the quantity n plus m.
	Nate:	Now what? Since we have x times y, and we want the log of that, we should use the definition of logs again.
3	**Voice-Over Nate:**	Now we have the log of the quantity x times y equals n plus m. Ok, we just have to remember how we started the proof. N equals log x and m equals log y. So substituting we get, log of the quantity x times y equals log of x plus log of y. Excellent! We have proven our property! Look at this connection. When you multiply numbers with exponents, you add the exponents. Similarly, when you take the log of a product, you add the logs.

	Nate:	Just like these manuals are updated with new information, we can update our math manual with this property. Our math manual is continually growing with what we learn and added to what we know. So lets see if using our updated math manual can help us prove one more property.
4	**Voice-Over Nate:**	The log of the quantity x to the n'th power is equal to n times the log of x. If you see log x to the n, you might quickly recognize it is the same as the log quantity x times x times x, taken n times. Using the previous property, we know that the log of x times x times x, n times, is equal to log of x plus log of x plus log of x, n times. Or, n times the log of x.
5	**Nate:**	Unfortunately, we cannot use this as a proof because this only makes sense for whole number values of n. In proofs, we have to choose strategies that can work for any value. But lets not get discouraged. We'll just consult our manual and try and again.
	Voice-Over Nate:	So, lets start the proof in a similar way to the first proof. Let m equal log of x. Using the definition of logarithm, 10 to the m equals x. Since we are trying to prove something about a power, lets raise both sides to the n'th power. Now we can use a Property of Exponents for powers, and get 10 to the quantity m times n equals x to the n. The property we are trying to prove has the log of x to the n'th power. So lets use the definition of logarithms again. M times n equals log of x to the n'th power. Great! At the beginning of the proof, we let m equal the log of x. By substituting we have n times of the log of x, equals log of x to the n! There, now I can add it to our math manual.
7	**Nate:**	I'm afraid it's going to take me a little longer to find my way around an engine. But now your math manual has grown, and you should have a pretty good understanding of why the properties of logarithms work. Problem solved. Want me to wash the windows or anything?

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