PROBLEM SOLVED
MAKING SENSE OF MATHEMATICS

Laws of Sines

Video Script

Scene		Full Transcript
1	**Skylar:**	Hey it's Skylar! Today, some friends of mine are taking a ride in a hot air balloon. I've been helping out at the launch site. Boy there sure is a lot that goes into getting a balloon aloft.
2	**Voice-Over Skylar:**	First, the balloon crew needs to find an open spot, free of trees and buildings for the launch point. The need plenty of room to lay out and inflate the envelope or bag of the balloon. Now that the balloon is inflated, they are ready to take off!
	Skylar:	It's a perfect day for a balloon ride and it's a perfect opportunity to discuss the Law of Sines. So get ready to go up, up and away and get another problem solved.
3	**Voice-Over Skylar:**	It was a beautiful launch! The chase crew has taken off after the hot air balloon and I'm staying here at the launch site. I'll catch up with them later. My friend Kyle is waiting at the landing point to help safely lower the balloon. I just spoke to my friend Doug who is up in the balloon taking photos. Doug just took a photo of Kyle from the landing point off in the distance. He's wondering how far the camera is away from Kyle. That's where the Law of Sines comes in. I'll show you.
	Skylar:	See how the launch point, the balloon and Kyle determine a triangle. We can find this distance by using the Law of Sines. First, start with what we know. Kyle is three miles from the launch site. Next, Kyle and I can find the angle of elevation using clinometer apps on our phones. The angle of elevation from the launch site is sixteen degrees. Kyle, what's your angle?
	Kyle:	The angle is eight degree.
4	**Voice-Over Skylar:**	The angle of elevation from his end is eight degrees. Ok, by subtracting the measures of the two angles from one-hundred eighty degrees, we can find the last angle is one-hundred fifty-six degrees.
	Skylar:	So, we know all three angles but only one side length. Is this enough information to help figure out the distance between the balloon and Kyle? To answer that, we need to ask, "What's the relationship between the lengths of

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		the sides of our triangles and the measure of its angles?" Frequently when we're trying to find a missing side length or angle measure of a triangle, a good approach is to use right triangles. Right triangles tend to be easier to deal with because we know a lot about them.
5	**Voice-Over Skylar:**	From studying right triangle trigonometry, you may recall, sine, cosine, and tangent. The sine of an angle is the ratio of the opposite side length, divided by the hypotenuse. Cosine is the adjacent side length divided by the hypotenuse. And tangent, is opposite over adjacent. Notice all of these functions are ratios of side lengths. Given any triangle, we can divide it into two right triangles. Then, using the sine ratio from earlier, we have, the sin A = h / c. And the sin C = h / a. Notice the height is a variable in both of these equations. If we solve for the height we get c times sin A = h and a times sin C = h. Since c times the sin of A equals the height, and a times the sin of C equals the height, we can set them equal to each other and we have c times the sin of A is equal to a times the sin of C. Rearranging the equation we get, the length of side c divided by the sin of angle C is equal to the length of side a divided by the sin of A. This is called the Law of Sines. Here's a cool thing! After all that algebra, H isn't even part of the equation anymore.
	Skylar:	The Law of Sines means that for **any** triangle, if you divide any side length by the sine of its opposite angle, and divide another side length by the sine of its opposite angle, you get the same value.
	Voice-over Skylar:	Notice that while the side lengths and the angles change, the ratios remain constant.
6	**Skylar:**	So, there's a relationship between the sins of the angle measures and the lengths of the opposite sides. What? Yeah, hold on. Doug is anxious for us to get back to our original problem. So lets see how this relationship can help us. We can use the Law of Sines to help find the distance between the balloon and Kyle.

Skylar: Remember, the landing point is three miles away from the starting point. As we found earlier, its opposite angle measures one-hundred fifty-six degrees. Now, setting up a proportion using the Law of Sines, we see that 3 / sin156 is equal to the distance to the landing d / sin16.

We use sixteen degrees because it's the angle opposite to the distance we're trying to find. Multiplying both sides by the sin16 we get, sin16 times 3 / sin156. Calculating this equation tells us that the picture was taken from two miles away.

Skylar: Two miles, yeah two! We used the Law of Sines! I think he's scared of heights.

Lets review. The Law of Sines is used to find angle measure and side lengths in triangles. So, when can we use it?

Voce-Over Skylar: You can use it when you know, an angle measure and its opposite side length plus: one additional angle measure or one additional side length.

The Law of Sines is a proportional relationship involving lengths and the sine of the opposite angle. A over the sin A (a / sin A) is equal to b over the sin B (b / sin B) and c over the sin C (c / sin C).

Skylar: See, your understanding of the Law of Sines is soaring to new heights. Now if we could just get Doug back on the ground safely. Problem solved.