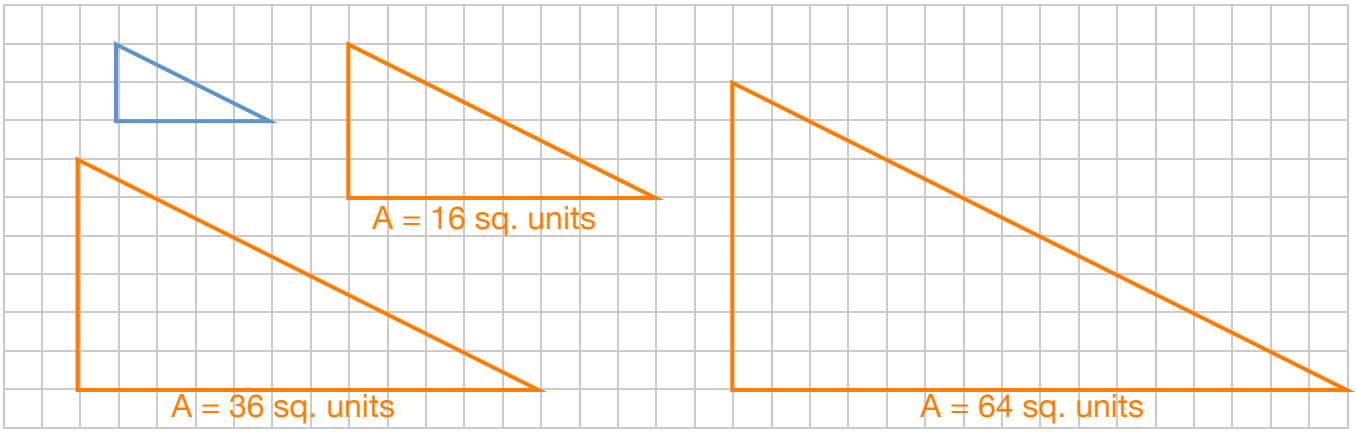
Changing Dimensions: Area
Extend Your Learning
Answers and Explanations

When you multiply the dimensions of a rectangle by n, the area of the new rectangle is n^2 times as large. When the scale factor is n, the area factor is n^2. Use the following problems to investigate if this relationship is true for other shapes.

1. Sketch three different triangles that are similar to the one shown below using the following directions.
   Triangle 1: Double the base and the height.
   Triangle 2: Triple the base and the height.
   Triangle 3: Quadruple the base and the height.



2. The area of the original triangle is 4 square units. Find the area of each triangle you drew.

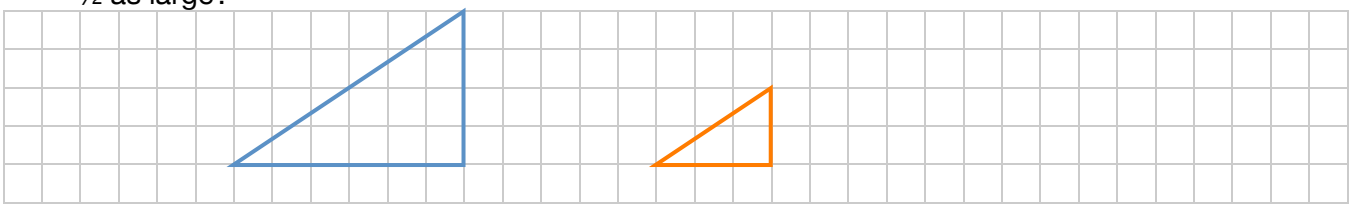
3. Describe how the area of a triangle changes when you double the dimensions.
   When you double the dimensions of a triangle, the area is four times as large.

4. Describe how the area of a triangle changes when you triple the dimensions.
   When you triple the dimensions of a triangle, the area is nine times as large.

5. Describe how the area of a triangle changes when you quadruple the dimensions.
   When you quadruple the dimensions of a triangle, the area is 16 times as large.

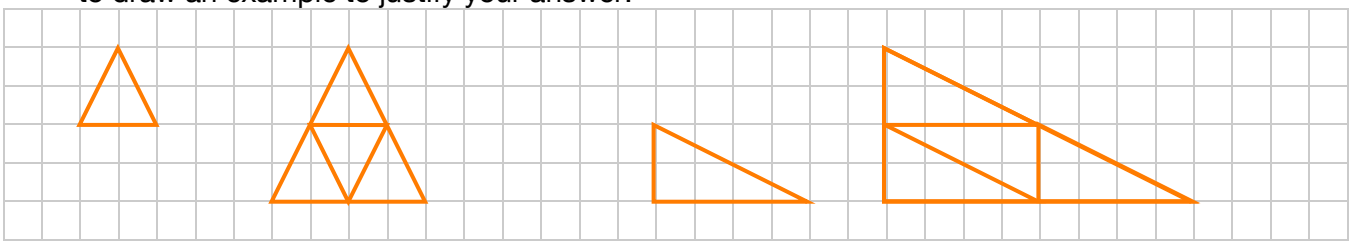
6. Describe how the area of a triangle changes when you make the dimensions n times as large.
   When you make the dimensions of a triangle n times as large, the area becomes n^2 times as large.

7. Sketch a triangle that is similar to the following triangle. Make the base and height ½ of the original base and height. What happens to the area of a triangle when you make the dimensions ½ as large?



The area of the larger triangle is 12 sq. units and the area of the smaller triangle is 3 sq. units. When you make the dimensions of a triangle ½ as large, the area is ¼ as large (12 ÷ 4 = 3). When the scale factor is ½, the area factor is $(½)^2$, or ¼.

8. Do you think this pattern will hold for all types of triangles? Why or why not? Use the grid below to draw an example to justify your answer.



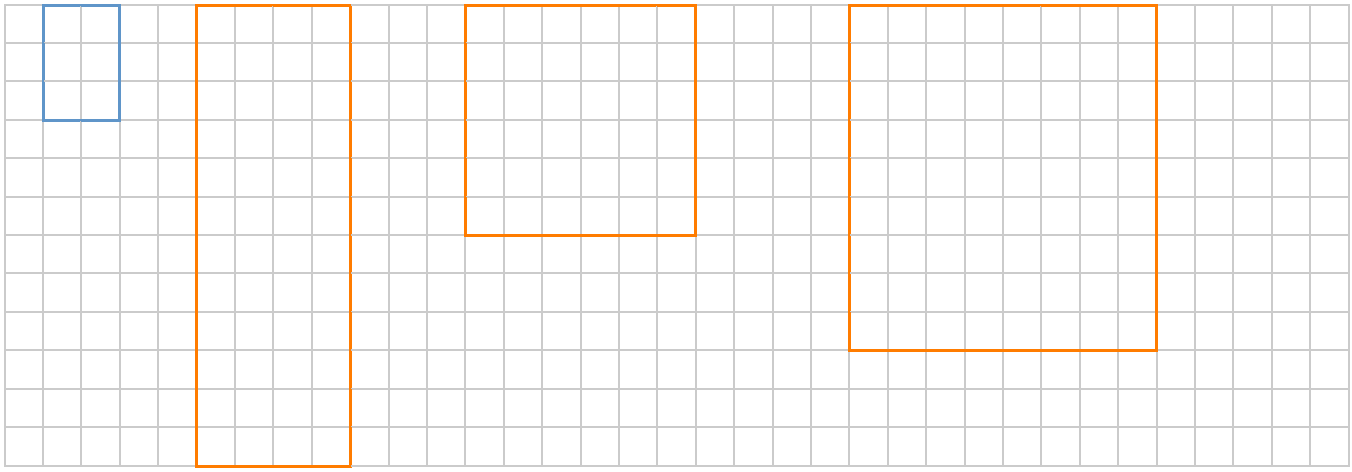
This pattern occurs for all types of triangles for the same reason that it occurs with all rectangles. This is because area is two-dimensional. The diagrams above show what happens to the area when you double the dimensions.

A second way to justify the pattern is to consider the area formula. The formula for area of a triangle is one half the base times the height (A = ½bh). Doubling both dimensions is the same as taking four times one half the base times the height, ½(2b)(2h) = 4(½bh). This is four times the original area.

9. The Cookie Factory has decided to make large round cookies for special events and holidays. They are trying to decide on a fair price for the cookies. A regular cookie has a radius of 2 in. and costs $0.85. The large cookie will have a radius of 6 in. What is a fair price?
When you triple the radius of a circle, the area is nine times as large. You can verify this by finding the area of each cookie ($A = \pi r^2$). The area of the small cookie is 4π sq. in. and the area of the large cookie is 36π sq. in. The price should be nine times the price of the smaller cookie, or $7.65 (9 • 0.85 = 7.65).

Use the following problems to investigate what happens if you multiply the two dimensions of a rectangle by different numbers.

10. Sketch three different rectangles that are similar to the one shown below using the following directions:
Rectangle 1: Make the base two times as long and the height four times as long.
Rectangle 2: Make the base three times as long and the height two times as long.
Rectangle 3: Make the base four times as long and the height three times as long.



11. How does the area of rectangle 1 compare to the area of the original rectangle?
The area of the original rectangle is 6 square units and the area of rectangle 1 is 48 square units. The area of rectangle 1 is 8 times as large as the area of the original rectangle (8 x 6 = 48).

12. How does the area of rectangle 2 compare to the area of the original rectangle?
The area of the original rectangle is 6 square units and the area of rectangle 2 is 36 square units. The area of rectangle 2 is 6 times as large as the area of the original rectangle (6 x 6 = 36).

13. How does the area of rectangle 3 compare to the area of the original rectangle?
The area of the original rectangle is 6 square units and the area of rectangle 3 is 72 square units. The area of rectangle 3 is 12 times as large as the area of the original rectangle (12 x 6 = 72).

14. What happens to the area of a rectangle when one dimension is n times as large and the other dimension is m times as large?
The area becomes (n times m) times as large as the original area.