**PROBLEM SOLVED**

MAKING SENSE OF MATHEMATICS

The video developed the formula for area of a trapezoid by forming a parallelogram with two congruent trapezoids. You will investigate two additional methods of developing the formula by completing the following problems.

Method 1: Cut out the trapezoid shown below. Cut along the dotted line and arrange the two pieces to form a parallelogram.


Rotate the top section of the trapezoid and fit it next to the bottom section of the trapezoid.
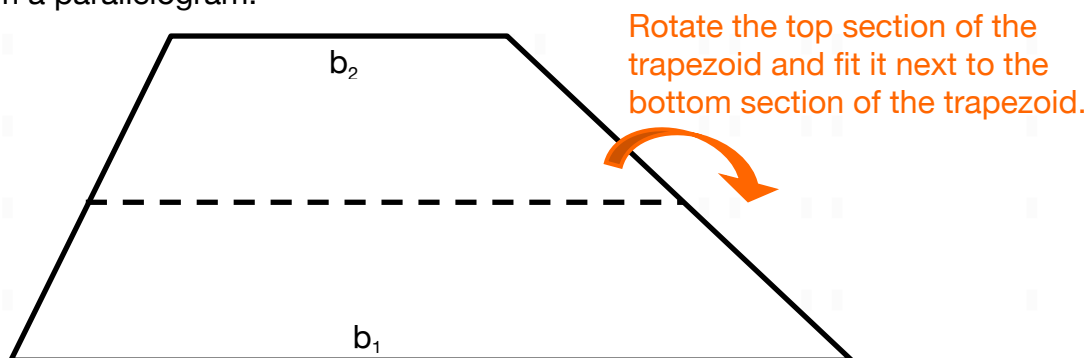
The resulting parallelogram looks like this:
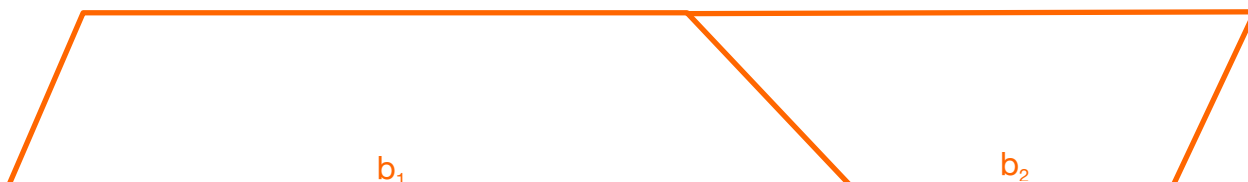
1. How does the area of the parallelogram compare to the area of the trapezoid?
    The trapezoid was cut into two parts and rearranged to form a parallelogram. No parts were added or discarded, so the parallelogram and the trapezoid have the same area.

2. How does the base of the parallelogram compare to the bases of the trapezoid?
    The base of the parallelogram is equal to the sum of the bases of the trapezoid $(b_1 + b_2)$.

3. How does the height of the parallelogram compare to the height of the trapezoid?
    The height of the parallelogram is equal to ½ the height of the trapezoid.

4. Use this representation to determine a formula for finding the area of any trapezoid. Explain your reasoning.
    The formula for area of a parallelogram is $A = bh$. The base of this parallelogram is the sum of the trapezoid bases $(b_1 + b_2)$. The height of this parallelogram is ½ the height of the trapezoid (½h). To find the area of this parallelogram multiply the base and the height $(b_1 + b_2)$(½h). This is equal to $½h(b_1 + b_2)$. This is also the area of the trapezoid.

Making Sense

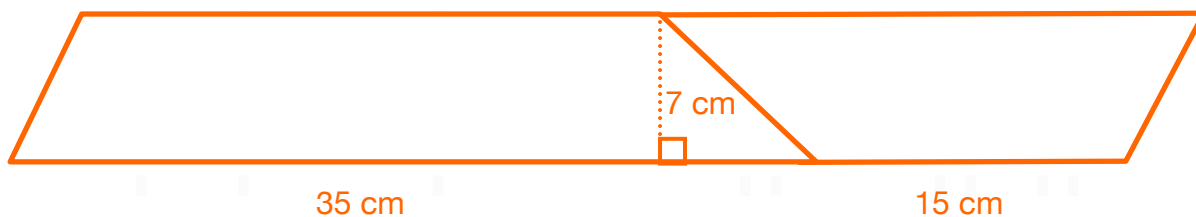**Method 2:** The trapezoid is divided into two different triangles.
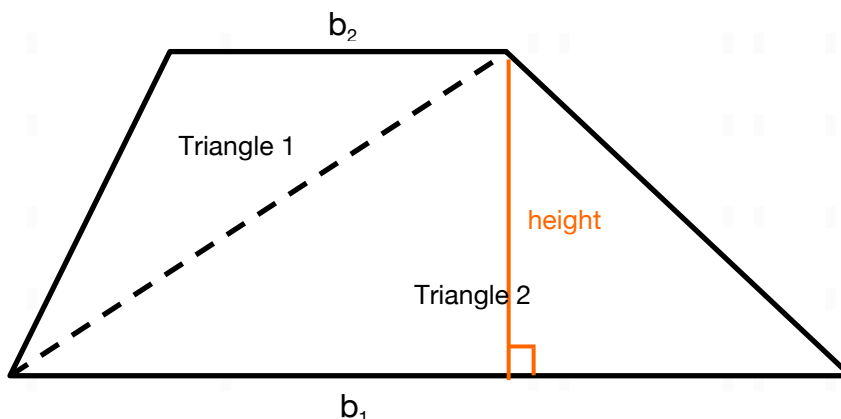
5. How does the area of the trapezoid compare to the area of the two triangles?
   The sum of the area of the two triangles is equal to the area of the trapezoid.

6. How does the height of the trapezoid compare to the height of each triangle?
   Triangle 1, Triangle 2, and the trapezoid all have the same height.

7. How do you find the area of Triangle 1? How do you find the area of Triangle 2?
   The formula for area of a triangle is $A = \frac{1}{2}bh$. The area of Triangle 1 is $\frac{1}{2}b_1h$. The area of Triangle 2 is $\frac{1}{2}b_2h$.

8. Use this representation to determine a formula for finding the area of any trapezoid. Explain your reasoning.
   The area of the trapezoid is equal to the sum of the areas of the triangles, so the area of the trapezoid is $\frac{1}{2}b_1h + \frac{1}{2}b_2h$. This is equal to $\frac{1}{2}h(b_1 + b_2)$.

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MAKING SENSE OF MATHEMATICS

In order to show that the results are the same with both methods, use each method to determine the area of the following trapezoid.

15 cm

14 cm

35 cm
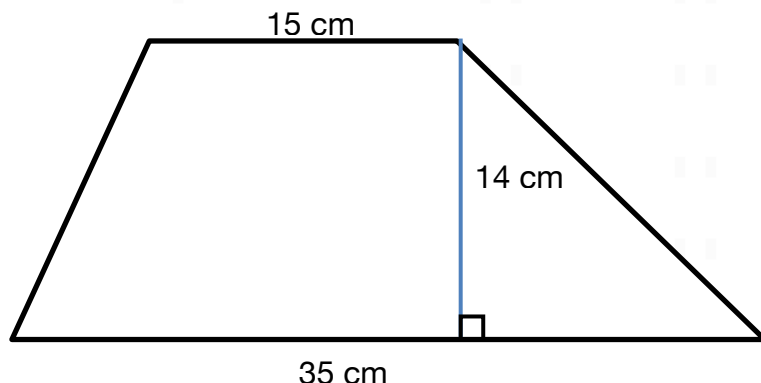
9. Use the reasoning from Method 1 to compute the area of the trapezoid. (Method 1: Cut the trapezoid into two parts and rearrange the parts to form a parallelogram.)
    The height of the parallelogram is ½ the height of the trapezoid (½ · 14 cm = 7 cm). The base of the parallelogram is equal to the sum of the bases of the trapezoid (35 cm + 15 cm = 50 cm). The area of the parallelogram is the base times the height (50 · 7 = 350 sq. cm). This is also the area of the trapezoid.

7 cm

35 cm   15 cm

10. Use the reasoning from Method 2 to compute the area of the trapezoid. (Method 2: Divide the trapezoid into two different triangles. Find the sum of the areas of the triangles.)
    The area of Triangle 1 is ½ · 15 · 14, or 105 sq. cm. The area of Triangle 2 is ½ · 35 · 14, or 245 sq. cm. The sum of the areas is 350 sq. cm. This is the area of the trapezoid.

15 cm

Triangle 1

Triangle 2

14 cm

35 cm

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