

After watching the *Changing Dimensions: Volume* video, make sense of the mathematics by reading through the problem situation and solution. Use the comments and questions in bold to help you understand how changing linear dimensions affects the area.

Problem: Carlos is helping Rick of Rathmell construction pour a cement patio. The volume tells Rick how much cement to order. He needs to determine the volumes of the old patio and the new patio.

How do you find the volume of any rectangular prism?

The volume of any rectangular prism is calculated by taking the length times the width times the height.

What is the volume of a patio measuring 4 feet by 3 feet by 2 feet?

CHANGING DIMENSIONS
VOLUME

$V = l \cdot w \cdot h$
 $= 4 \text{ ft.} \cdot 3 \text{ ft.} \cdot 2 \text{ ft.}$
 $= \mathbf{24 \text{ cu. ft.}}$

4 ft. 3 ft. 2 ft.

(i talked w/ jon... how the cubes come in)

What is the volume of the old patio?

Old patio measurements: **12 feet long by 10 feet wide by 3 inches high**
 (Recall 3 inches = $\frac{1}{4}$ ft.)

CHANGING DIMENSIONS
VOLUME

$V = l \cdot w \cdot h$
 $= 12 \text{ ft.} \cdot 10 \text{ ft.} \cdot \frac{1}{4} \text{ ft.}$
 $= \mathbf{30 \text{ cu. ft.}}$

12 ft. 10 ft. $\frac{1}{4}$ ft.

If the linear dimensions of the old patio are each doubled, what is the volume of the new patio?
 New patio measurements: 24 feet long by 20 feet wide by 6 inches high
 (Recall 6 inches = $\frac{1}{2}$ ft.)

CHANGING DIMENSIONS
VOLUME

$V = l \cdot w \cdot h$
 $= 24 \text{ ft.} \cdot 20 \text{ ft.} \cdot \frac{1}{2} \text{ ft.}$
 $= 240 \text{ cu. ft.}$

24 ft. 20 ft. $\frac{1}{2}$ ft.

Why is the volume of the new patio eight times the volume of the old patio instead of doubled when the linear dimensions doubled? To investigate, look at some quick examples.

Example 1

If the linear dimensions of a rectangular prism are doubled (scale factor 2), what are the dimensions of the new rectangular prism?

original prism: 2 ft. by 1 ft. by 1 ft.

new prism: 4 ft. by 2 ft. by 2 ft.

If the linear dimensions of a rectangle are doubled (scale factor 2), what is the relationship between the volume of the original prism and the volume of the new prism?

Scale Factor: 2
Volume Factor: 8

$V = l \cdot w \cdot h$
 $V = 2 \text{ ft.} \cdot 1 \text{ ft.} \cdot 1 \text{ ft.}$
 $V = 2 \text{ cu. ft.}$

CHANGING DIMENSIONS
VOLUME

$V = 2 \text{ cu. ft.}$

$\times 8$

$V = 16 \text{ cu. ft.}$

$V = l \cdot w \cdot h$
 $V = 4 \text{ ft.} \cdot 2 \text{ ft.} \cdot 2 \text{ ft.}$
 $V = 16 \text{ cu. ft.}$

The new volume is 8 times larger than the original, so the volume factor is 8.

Example 2

If the linear dimensions of a rectangular prism are tripled (scale factor 3), what are the dimensions of the new rectangular prism?

original prism: 2 ft. by 1 ft. by 1 ft.

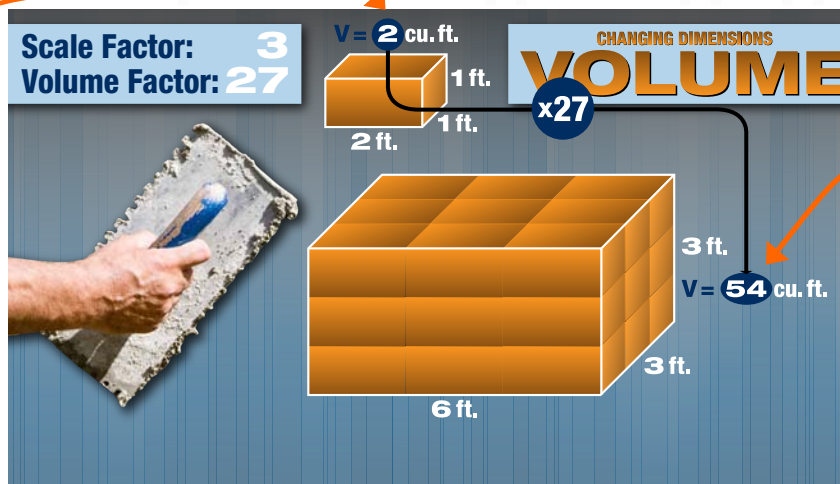
new prism: 6 ft. by 2 ft. by 2 ft.

If the linear dimensions of a rectangle are tripled (scale factor 3), what is the relationship between the volume of the original rectangular prism and the volume of the new rectangular prism?

$$V = l \cdot w \cdot h$$

$$V = 2 \text{ ft.} \cdot 1 \text{ ft.} \cdot 1 \text{ ft.}$$

$$V = 2 \text{ cu. ft.}$$



$$V = l \cdot w \cdot h$$

$$V = 6 \text{ ft.} \cdot 2 \text{ ft.} \cdot 2 \text{ ft.}$$

$$V = 54 \text{ cu. ft.}$$

The new volume is 27 times larger than the original, so the volume factor is 27.

Example 3

If the linear dimensions of a rectangular prism are multiplied by 4 (scale factor 4), what are the dimensions of the new rectangular prism?

original prism: 2 ft. by 1 ft. by 1 ft.

new prism: 8 ft. by 4 ft. by 4 ft.

If the linear dimensions of a prism are multiplied by 4 (scale factor 4), what is the relationship between the volume of the original prism and the volume of the new prism?

$V = l \cdot w \cdot h$
 $V = 2 \text{ ft.} \cdot 1 \text{ ft.} \cdot 1 \text{ ft.}$
 $V = 2 \text{ cu. ft.}$

Scale Factor: 4
Volume Factor: 64

$V = 2 \text{ cu. ft.}$
 $V = 128 \text{ cu. ft.}$

CHANGING DIMENSIONS
VOLUME

$\times 64$

$V = l \cdot w \cdot h$
 $V = 8 \text{ ft.} \cdot 4 \text{ ft.} \cdot 4 \text{ ft.}$
 $V = 128 \text{ cu. ft.}$

The new volume is 64 times larger than the original, so the volume factor is 64.

Study the following table and look for a pattern. What is the relationship between the scale factor and the volume factor?

Scale Factor	Volume Factor
2	4
3	9
4	16
n	

Looking at the pattern above, as each of the dimensions of a prism is multiplied by the same number, the volume is multiplied by the cube of that number. Whatever the scale factor is, the volume factor is the cube of that number.

Scale Factor	Volume Factor
2	$8 = 2^3$
3	$27 = 3^3$
4	$64 = 4^3$
n	n^3