PROBLEM SOLVED
MAKING SENSE OF MATHEMATICS

Inverse Proportionality

A Closer Look at the Video

After watching the *Inverse Proportionality* video, make sense of the mathematics by taking a closer look at the problem situations and solutions. Use the comments and questions in bold to help you solve the problems and develop a deeper understanding of inverse proportions.

Inverse proportionality can help determine the time it will take to get a job done. Look at the relationship between the number of machines needed and the number of days it takes to complete a job. If you have half as many machines, the job will take twice as long. There is an inverse relationship between the number of machines used and the amount of time it takes to complete the job. As one quantity increases, the other quantity decreases proportionally. The more machines used, the less time is needed to complete the job.

Problem 1: Qualitek is a company that makes industrial pins. There are usually six machines available, and the six machines can make a total of 300 pins in eight days. The company received an order for 300 industrial pins, but one half of the machines are down for maintenance. How long will it take to complete the job with the three available machines?

Study the following table. How many days will it take to produce 300 pins if there is only one half the number of machines? Will it take more or less than eight days? Explain your answer.

Number of machines	Number of pins	Number of days
6	300	8
3	300	?

If you have one half the number of machines, it will take twice as long to produce the same number of pins. This is an inverse relationship. The inverse of $\frac{1}{2}$ is $\frac{2}{1}$ or 2. To find the number of days needed to complete this job with only three machines, multiply the number of days, 8, by 2. It will take 2 x 8 or 16 days to produce 300 pins using only three machines.

Number of machines	Number of pins	Number of days
x $\frac{1}{2}$ 6	300	8 x 2
3	300	16

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Problem 2: Qualitek received a new order for 1,500 industrial pins, but two of the three machines are scheduled for use on another project. How long will it take to complete the job with only one machine?

Break the problem into two parts. First, determine the number of days it will take three machines to produce 1500 pins. Use the following table to show your thinking.

Number of machines	Number of pins	Number of days
3	300	16
3	1500	?

There is a direct relationship between the number of pins produced and the number of days it takes to produce the pins. As the number of pins increases, the number of days required to produce them also increases. One thousand five hundred is five times as many pins as 300, so it will take five times as many days to produce the 1500 pins. Since 5 x 16 = 80, it would take 80 days for three machines to produce 1500 pins.

Number of machines	Number of pins	Number of days
3	x 5 300	16 x 5
3	1500	80
1	1500	?

Now determine the number of days it will take one machine to produce 1500 pins. There is an inverse relationship between the number of machines and the number of days required. The fewer machines you have, the more days it will take. If you have one third the number of machines, it will take three times as long to produce the same number of pins. The inverse of one third is three. To find the number of days needed to complete this job with only one machine, multiply the number of days, 80, by 3. It will take 3 x 80 or 240 days to produce 1500 pins using only one machine.

Number of machines	Number of pins	Number of days
3	300	16
x $\frac{1}{3}$ 3	1500	80 x 3
1	1500	240

Problem 3: The customer who ordered 1,500 industrial pins needs the job completed in 20 days. How many machines are required to complete the job in 20 days?

In problem 2, you found it would take 240 days to produce 1500 industrial pins using only one machine. Use a table to help you determine the number of machines needed to complete the job in 20 days.

Number of machines	Number of pins	Number of days
1	1500	240
?	1500	20

Rather than 240 days, the company has 20 days. What fraction of 240 is 20? Since 240 divided by 20 is 12, one twelfth of 240 is 20. **If the company wants to produce 1500 pins in one twelfth the number of days, how many machines are needed?** There is an inverse relationship between the number of days and the number of machines needed to complete this job. The inverse of one twelfth is 12. If you have one twelfth the number of days to produce the same number of pins, it will take twelve times as many machines. It will take 12 x 1, or 12 machines to produce 1500 pins in 20 days.

Number of machines	Number of pins	Number of days
x 12 1	1500	240 $\times \frac{1}{12}$
?	1500	20

An inverse relationship occurs in a situation where one quantity increases while the other quantity decreases proportionally. Keep the following ideas in mind as you solve problems involving inverse proportionality.

- Tables are useful in solving problems involving inverse relationships.
- When solving problems involving inverse relationships, one quantity is multiplied by a number and the other quantity is multiplied by the inverse of the number.
- The inverse of a number is the reciprocal of the number. The following pairs of numbers are inverses of each other:
  $\frac{1}{2}$ and $\frac{2}{1}$ or 2 $\quad$ $\frac{1}{3}$ and $\frac{3}{1}$ or 3 $\quad$ $\frac{1}{12}$ and $\frac{12}{1}$ or 12

A direct relationship occurs in a situation where two quantities both increase proportionally. For more information on directly proportional relationships see the following videos:

- Equal Ratios
- Graphing Proportional relationships
- Scale Factor
- Similarity: Using Proportions