Exponential Growth

A Closer Look at the Video

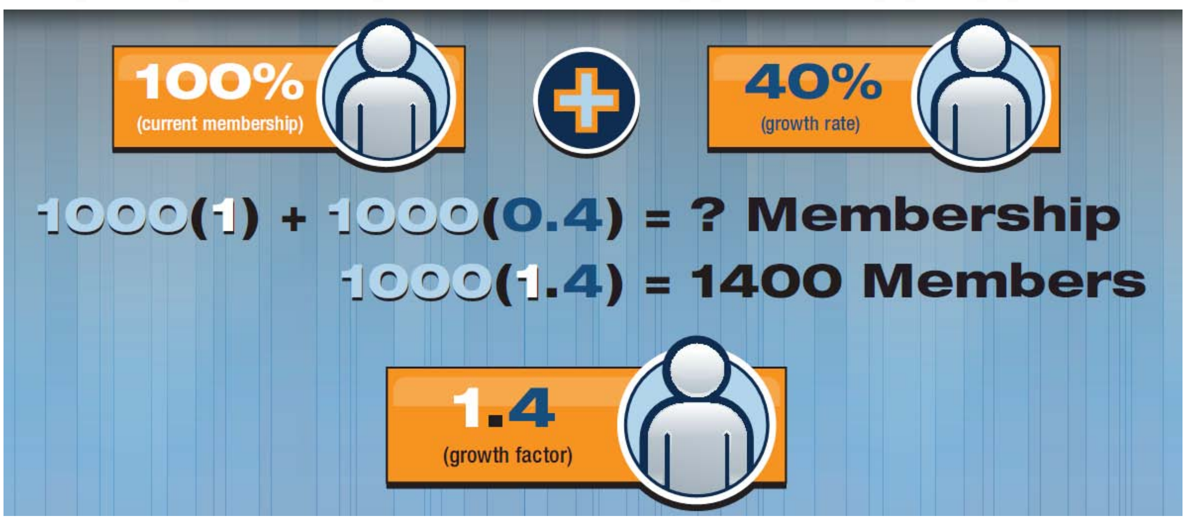
After watching the *Exponential Growth* video, make sense of the mathematics by taking a closer look at the problem situations and solutions. Use the comments and questions in bold to help you investigate the key points of the video and develop a deep understanding of exponential growth.

Problem: The co-founders of *Live Link Tickets* have developed a networking service that allows parents to go online and watch their child's concert, play, or sporting event live, no matter where they are in the world! Membership is growing like crazy! In fact, they already have one thousand members. Answer the questions below to help them determine how many servers they'll need to accommodate future expansion.

The *Live Link Tickets* company is growing at a rate of 40% per month. What does that mean?
It means that next month's membership will be 100% of the members, plus an additional 40% of the members. That is 140% of the original number of members.

How can you write an equation to find the number of members at the end of one month?
The current membership is equal to 1000•100% or 1000•1. New membership is equal to 1000•40% or 1000•0.4. The total number of members at the end of Month 1 is equal to the sum of the current membership and the new membership. This sum is equal to 1000•140% or 1000•1.4.

One and four tenths is called the growth factor and is equal to one plus the growth rate.



How can we use the growth factor of 1.4 to find the number of members at the end of Month 2?
We take the membership of Month 1, multiply it by the growth factor of 1.4, and get 1,960 members by the end of Month 2.

How can we use the growth factor of 1.4 to find the number of members at the end of Month 3?
If we continue the process of multiplying our new month's membership by the growth factor of 1.4, we predict 2,744 members by the end of Month 3.

# of months (n)	total membership (M)	
0	1000	
1	1400	x 1.4
2	1960	x 1.4
3	2744	x 1.4

Notice that each month's membership is not found by adding the same amount each month, but by **multiplying the previous month's membership by the growth factor.** The team could predict the membership in 1 year by continuing to multiply by 1.4 for each of the 12 months.

What is another way we can predict the membership after one year?
Since we repeatedly multiply by the growth factor, we can use exponents to quickly calculate the predicted membership for any month.

# of months (n)	total membership (M)	
0	1000	1000
1	1000(1.4)	1400
2	1000(1.4)(1.4)	1960
3	1000(1.4)(1.4)(1.4)	2744

The exponent corresponds to the number of months. To calculate the membership by the end of Month *n,* we take $1{,}000(1.4)^n$. So, to predict the membership by the end of Year 1, we calculate $1{,}000(1.4)^{12}$, which is 56,694 members.

# of months (n)	total membership (M)		
0	1000	$= 1000(1.4)^0$	1000
1	1000(1.4)	$= 1000(1.4)^1$	1400
2	1000(1.4)(1.4)	$= 1000(1.4)^2$	1960
3	1000(1.4)(1.4)(1.4)	$= 1000(1.4)^3$	2744
n	$1000\underbrace{(1.4)\ldots(1.4)}_{n}$	$= 1000(1.4)^n$	

How can we calculate growth for any situation?
We need two important values:

- the value of the dependent variable when n equals 0, which can be represented by "*a*"
- the growth factor, which can be represented by *1 + r*

What are the two important values for our problem?

- "a" is the value of the dependent variable when n = 0
    - In our equation, this value is 1,000 since $1000(1.4)^0$=1000.
- the growth factor
    - In our equation, the growth factor is 1.4 or one plus the growth rate.

How can we generalize our exponential growth equation?
Instead of using M for membership and n for number of months, we can replace these variables with "y" and "x". So any exponential growth equation can be represented by $y = a(1+r)^x$.