PROBLEM SOLVED
MAKING SENSE OF MATHEMATICS

Comparing Fractions Mentally
A Closer Look at the Video

After watching the *Comparing Fractions Mentally* video, make sense of the mathematics by taking a closer look at the problem situations and solutions. Use the comments and questions in bold to help you compare fractions mentally and solve the problems.

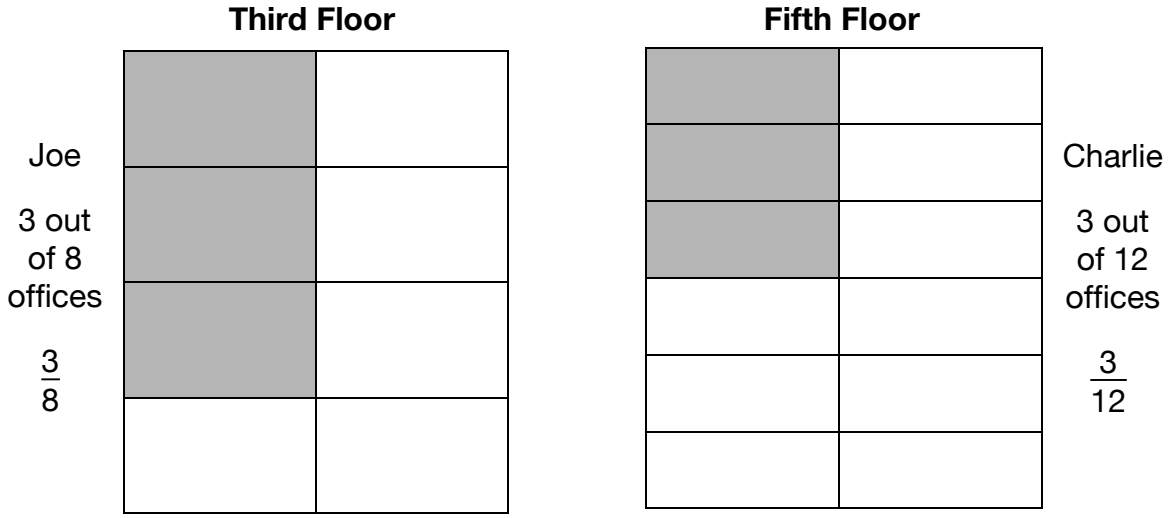
One way to compare fractions is by finding a common denominator. This is a useful procedure, but it is not always the most efficient. The video presents three problems and three strategies for comparing fractions without finding a common denominator. Use the diagrams to help you make sense of each strategy.

Problem 1: Joe and Charlie are cleaning offices in the same building. Joe cleans three eighths of the third floor in the same amount of time that Charlie cleans three twelfths of the fifth floor. The floors are the same size. Who is the faster cleaner?

To decide who is the faster cleaner, you need to know which fraction is greater. What do you notice about the fractions in this problem? What is the same and what is different? The fractions have the same numerator but different denominators.

What do the numerators represent? The numerators represent the number of offices that Joe and Charlie cleaned. Joe cleaned three out of eight offices and Charlie cleaned three out of twelve offices.

What do the denominators represent? The denominators represent the total number of offices on each floor. Joe cleaned a floor divided into eight equal-sized offices and Charlie cleaned a floor that is the same size, but divided into twelve equal-sized offices.



MakingSense

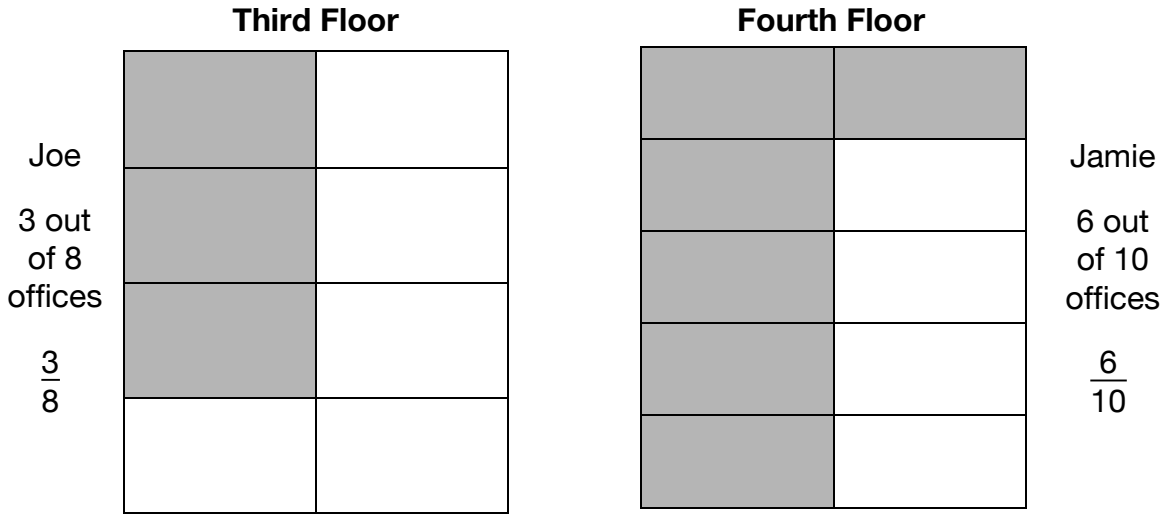
The denominator determines the size of each part. Which is greater, eighths or twelfths? Explain your reasoning. Eighths are larger than twelfths. The fraction with the smaller denominator has larger parts.

How can you use this information to decide if $\frac{3}{8}$ or $\frac{3}{12}$ is greater? Both fractions have three parts, but eighths are larger than twelfths. Three larger parts are greater than three smaller parts, so $\frac{3}{8}$ is greater than $\frac{3}{12}$. This means Joes cleans faster than Charlie.

Strategy 1: If fractions have the same numerator, compare the denominators to determine which fraction is greater.

Problem 2: In the same amount of time that Joe cleans three eighths of the third floor, Jamie cleans six tenths of the fourth floor. The floors are the same size. Who is the faster cleaner?

You do not need to find a common denominator to answer this question. Notice that both fractions are close to one half.



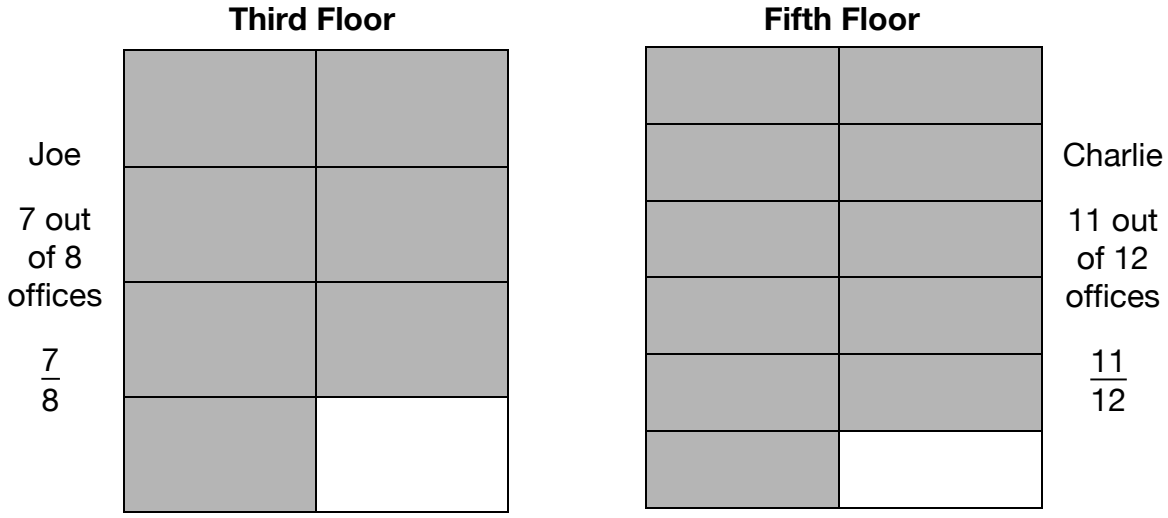
Joe cleaned $\frac{3}{8}$ of third floor. Is $\frac{3}{8}$ more or less than one half? Since $\frac{4}{8}$ is equal to $\frac{1}{2}$, $\frac{3}{8}$ is less than $\frac{1}{2}$. **Jamie cleaned $\frac{6}{10}$ of the fourth floor. Is $\frac{6}{10}$ more or less than one half?**

Since $\frac{5}{10}$ is equal to $\frac{1}{2}$, $\frac{6}{10}$ is greater than $\frac{1}{2}$. Since $\frac{3}{8}$ is less than $\frac{1}{2}$ and $\frac{6}{10}$ is greater than $\frac{1}{2}$, $\frac{6}{10}$ must be greater than $\frac{3}{8}$. This shows that Jamie cleans faster than Joe.

Strategy 2: Check if the fractions are more or less than a benchmark fraction, such as one half, to determine which is greater.

Problem 3: In the same amount of time that Joe cleans seven eighths of the third floor, Charlie cleans eleven twelfths of the fifth floor. The floors are the same size. Who is the faster cleaner?

Both fractions are close to one. How can you determine which fraction is greater by comparing the fractions to one?



Joe cleaned $\frac{7}{8}$ of third floor, so he has $\frac{1}{8}$ left to clean. Charlie cleaned $\frac{11}{12}$ of fifth floor, so he has $\frac{1}{12}$ left to clean. Since $\frac{1}{12}$ is less than $\frac{1}{8}$, $\frac{11}{12}$ is closer to one than $\frac{7}{8}$. This means Charlie cleaned more and is the faster cleaner.

Strategy 3: Compare fractions to the benchmark of one to determine which is greater.

Finding a common denominator is an accurate and useful procedure, but not always the most efficient. Students who know a variety of reasoning strategies for ordering fractions can flexibly approach problems and solve problems accurately and efficiently.