Standard Deviation
Use What You've Learned
Answers and Explanations

After watching the video, *Standard Deviation*, complete the following problems.

1. Let $x_1 = 2$, $x_2 = 2$, $x_3 = 3$, $x_4 = 6$, $x_5 = 6$. The summation notation is used in the formula for standard deviation. Become more familiar with this notation by evaluating the following expressions.

   a. $\sum_{i=1}^{5} x_i$

   $\sum_{i=1}^{5} x_i = x_1 + x_2 + x_3 + x_4 + x_5$

   $\sum_{i=1}^{5} x_i = 2 + 2 + 3 + 6 + 6$

   $\sum_{i=1}^{5} x_i = 19$

   b. $\sum_{i=2}^{4} x_i$

   $\sum_{i=2}^{4} x_i = x_2 + x_3 + x_4$

   $\sum_{i=2}^{4} x_i = 2 + 3 + 6$

   $\sum_{i=2}^{4} x_i = 11$

   c. $\sum_{i=1}^{5} (x_i)^2$

   $\sum_{i=1}^{5} (x_i)^2 = (x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2 + (x_5)^2$

   $\sum_{i=1}^{5} x_i = (2)^2 + (2)^2 + (3)^2 + (6)^2 + (6)^2$

   $\sum_{i=1}^{5} x_i = 4 + 4 + 9 + 36 + 36$

   $\sum_{i=1}^{5} x_i = 89$

d. $\left(\sum_{i=1}^{5} x_i\right)^2$

$(\sum_{i=1}^{5} x_i)^2 = (x_1 + x_2 + x_3 + x_4 + x_5)^2$

$(\sum_{i=1}^{5} x_i)^2 = (2 + 2 + 3 + 6 + 6)^2$

$(\sum_{i=1}^{5} x_i)^2 = (19)^2$

$(\sum_{i=1}^{5} x_i)^2 = 361$

e. $\sum_{i=1}^{5} (x_i - 3)^2$

$\sum_{i=1}^{5} (x_i - 3)^2 = (x_1 - 3)^2 + (x_2 - 3)^2 + (x_3 - 3)^2 + (x_4 - 3)^2 + (x_5 - 3)^2$

$\sum_{i=1}^{5} (x_i - 3)^2 = (2 - 3)^2 + (2 - 3)^2 + (3 - 3)^2 + (6 - 3)^2 + (6 - 3)^2$

$\sum_{i=1}^{5} (x_i - 3)^2 = 1 + 1 + 0 + 9 + 9$

$\sum_{i=1}^{5} (x_i - 3)^2 = 20$

2. Every morning, I get out of bed and do sit-ups until I give up. Here is how it went last week:

Monday	140
Tuesday	179
Wednesday	150
Thursday	197
Friday	150
Saturday	40
Sunday	138

a. What is my mean number of sit-ups?

$\frac{140 + 179 + 150 + 197 + 150 + 40 + 138}{7} = 142$

b. What is my standard deviation?

x_i	$x_i - \bar{X}$	$(x_i - \bar{X})^2$
140	140-142	$(-2)^2=4$
179	179-142	$(37)^2=1369$
150	150-142	$(8)^2=64$
197	197-142	$(55)^2=3025$
150	150-142	$(8)^2=64$
40	40-142	$(-98)^2=9604$
138	138-142	$(-4)^2=16$

$$\sum (x - \bar{X})^2 = 14{,}146$$

$$\frac{\sum (x - \bar{X})^2}{n-1} = \frac{14146}{6}$$

$$\frac{\sum (x - \bar{X})^2}{6} \approx 2020.86$$

$$\sqrt{\frac{\sum (x - \bar{X})^2}{6}} \approx 48.56$$

c. I don't think Saturday should count, because I was interrupted when my kitten jumped on me. If we eliminate Saturday from our data, what is my new standard deviation?

x_i	$x_i - \bar{X}$	$(x_i - \bar{X})^2$
140	140-159	$(-19)^2=361$
179	179-159	$(20)^2=400$
150	150-159	$(-9)^2=81$
197	197-159	$(38)^2=1444$
150	150-159	$(-9)^2=81$
138	138-159	$(-21)^2=441$

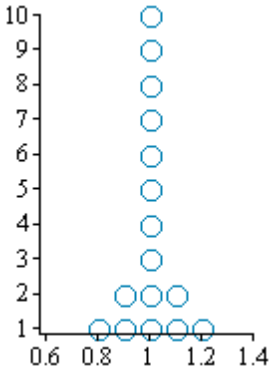
$$\sum (x - \bar{X})^2 = 2808$$

$$\frac{\sum (x - \bar{X})^2}{n-1} = \frac{2808}{5}$$

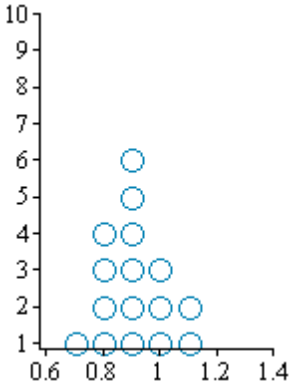
$$\sqrt{\frac{\sum (x - \bar{X})^2}{5}} \approx 23.6981$$

d. Your answer to part c should have been smaller than your answer to part b. Why should we have known, before doing any calculations, that removing Saturday's result would cause the standard deviation to go *down*?
40 is significantly less than the mean of 142. Standard deviation measures how much the data differs from the mean. So, removing this outlier will make the standard deviation of the remaining days much smaller.

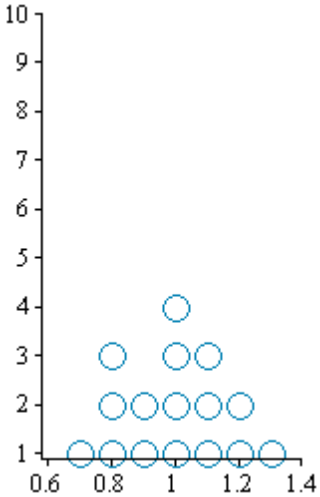
3. A government inspector has been assigned to four bakeries: Acme Bakers, Buns R Us, Croissant Your Heart, Do Do Donuts. Over a period of several months, she buys several "one-pound" loaves from each bakery and measures them precisely. Here are her results:



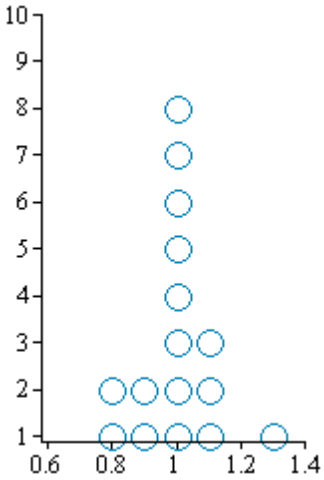
Acme Baker



Buns R Us



Croissant your Heart



Do Do Donuts

MakingSense

a. Estimate the mean weight of a "one-point loaf" from each bakery.
A, C, D: 1 B:0.9 Answers may vary.

b. Order the bakeries by standard deviation from lowest to highest.
A, D, B, C

c. Which one of the bakeries most likely uses a machine to measure their dough? Explain your reasoning.
A

Since machines are more consistent in measuring dough, the bakery with the lowest standard deviation probably uses a machine.

d. Which one of the bakeries most likely measures their dough by hand? C
Why? Machines are more consistent than humans, so human-measured dough will have a higher standard deviation.

e. The inspector issues a fine to one of the bakeries. Which bakery, and why?
B. Their mean loaf weight is 0.9, which is less than one pound. So they are, on average, giving their customers less bread than advertised.

4. Create a data set that contains six values, which has a mean of 10 and a standard deviation of zero.
10, 10, 10, 10, 10, 10

5. Create a data set that contains six values, has a mean of 0 and a standard deviation of one hundred.
Answers may vary. One possible solution: 100, 100, 100, -100, -100, -100