Mean, Median, Mode (Part 2)

Use What You've Learned
Answers and Explanations

1. The following table shows the number of inches of rain received each day for seven days, but the amount of rain is recorded incorrectly on one of the days. The mean amount of rain for the week is one inch per day.

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Rainfall	1.5 in.	2 in.	2 in.	0 in.	0 in.	10 in.	0.5 in.

Which day do you think is recorded incorrectly? Why?
Strategy 1: The amount of rain recorded for Friday is much larger than the amount recorded for any other day, and 10 inches is an extremely large amount for many regions of the world. For these reasons, the amount of rain recorded for Friday seems to be incorrect.

Strategy 2: The amount of rain recorded for Friday must be incorrect. If you take the amount of rain recorded for Friday, 10 inches, and spread it out over 7 days, the mean amount per day will be over 1 inch.

What should be the amount of rainfall for the day that is recorded incorrectly? Explain how you determined your answer and justify why it is correct.
Strategy 1: If the mean amount of rainfall for seven days is 1 inch, the total amount of rain for the seven days must equal 7 inches. The total amount of rain without Friday is 6 inches, so it must have rained 1 inch on Friday.

Strategy 2: Equalize the rainfall among the days, so each day equals the mean, or 1 inch. Subtract 0.5 inch from Sunday and add it to Saturday. Next, subtract 1 inch from both Monday and Tuesday in order to add 1 inch to both Wednesday and Thursday. In order for each day to equal the mean, 1 inch, it must have rained 1 inch on Friday.

If all seven data values in the table were originally recorded correctly, what would be the mean?
If it did rain 10 inches on Friday, the mean amount of rainfall would equal approximately 2.29 inches, or about $2\frac{1}{4}$ inches ($1.5 + 2 + 2 + 0 + 0 + 10 + 0.5 = 16$ and $16 \div 7 \approx 2.29$).

2. Maggie has taken five 10-point mathematics quizzes this year. The mean of her quiz scores is 8. She takes a sixth quiz, and her mean is now 7. What did she score on the last quiz?
Strategy 1: If the mean score of the first five quizzes is 8, the total points scored on those five quizzes must be 5 x 8, or 40. If the mean score of six quizzes is 7, the total points scored on those six quizzes must be 6 x 7, or 42. If Maggie's quiz total was 40 before the sixth quiz and 42 after the sixth quiz, Maggie must have scored 42 – 40, or 2 points, on the sixth quiz.

Strategy 2: If the mean of five scores is 8, each score could equal 8. If the mean of six scores is 7, each score could equal 7. In order for five scores of 8 to change to six scores of 7, add 7 and subtract one from each 8. Adding 7 and subtracting 5 is the same as adding 2 to the total, so Maggie scored 2 points on the last quiz.

$$\begin{array}{cccccc} 8 & 8 & 8 & 8 & 8 & \\ -1 & -1 & -1 & -1 & -1 & +7 \\ \hline 7 & 7 & 7 & 7 & 7 & 7 \end{array} = 2$$

3. Zaid has taken seven 10-point math quizzes this semester and earned the following scores: 10, 7, 7, 6, 9, 8, 9. What is his mean quiz score? What does he need on the eighth quiz to have his mean equal 9? Explain your reasoning.
The mean score of the first seven quizzes is 8 (10 + 7 + 7 + 6 + 9 + 8 + 9 = 56 and 56 ÷ 7 = 8). Zaid needs a total of 72 points on eight quizzes in order to have a mean score of 9 (8 x 9 = 72). This means Zaid needs to earn 16 points on the eighth quiz. (72 – 56 = 16) This is not possible without extra credit. If Zaid earns 10 points on the eighth quiz, his total will equal 66 points and his mean score will be 8.25. (56 + 10 = 66 and 66 ÷ 8 = 8.25)

4. Alec took five 10-point mathematics quizzes this year. The mean of his scores is 8, the mode is 7, and the median is 7. What are his five scores?
Strategy 1: Start by representing Alec's five scores in ascending order with five blank lines. If the median score is 7, the middle number must be 7.

___ ___ **7** ___ ___

If the mean score is 8, the total of all five scores must equal 5 x 8, or 40.

___ + ___ + **7** + ___ + ___ **= 40**

If the mode score is 7, at least two of the scores must equal 7 and total 14. The remaining three scores must total 40 – 14, or 26. A score cannot be greater than the total number of points each quiz is worth, or 10 points. The following four sets of whole numbers equal 26 and do not include a number greater than 10.

10 + 10 + 6 **10 + 9 + 7** **10 + 8 + 8** **9 + 9 + 8**

In order for 7 to be the middle number, at least one of the three remaining scores must be equal to or less than 7. This eliminates two sets of numbers. There is only one mode, so our set cannot include two scores equal to 10. Only one set of whole numbers meets all of the requirements of the problem.

7 + 7 + 7 + 9 + 10 = 40