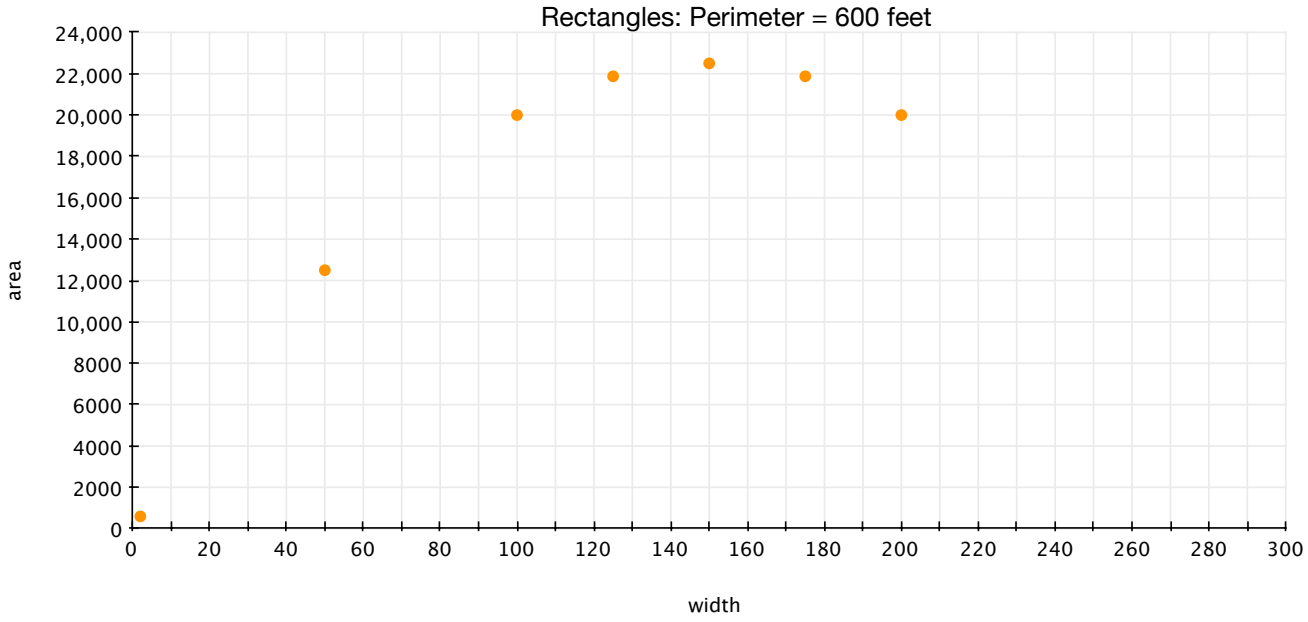
Fixed Perimeter, Changing Area

Use What You've Learned
Answers and Explanations

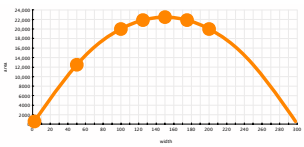
1. Mia purchased an underground dog fence that includes 500 feet of wire. Mia also bought an additional 100 feet of wire. She wants to fence in a rectangular region using all of the wire.
   a. What dimensions could Mia use to construct her fence? What is the area of each region? There are many possible solutions to this problem. List at least six solutions in the table below. There are seven solutions in the table. Many more solutions are possible.

Rectangles: Perimeter = 600 feet		
Width	Length	Area
2 ft.	298 ft.	596 sq. ft.
50 ft.	250 ft.	12,500 sq. ft.
100 ft.	200 ft.	20,000 sq. ft.
125 ft.	175 ft.	21,875 sq. ft.
150 ft.	150 ft.	22,500 sq. ft.
175 ft.	125 ft.	21,875 sq. ft.
200 ft.	100 ft.	20,000 sq. ft.

   b. Use the grid below to graph the widths and areas of rectangles that have a perimeter of 600 feet. Make the x-axis equal to the width of the rectangle and the y-axis equal to the area of the rectangle. The graph shows seven solutions. Many more solutions are possible.



   c. Describe what the graph would look like if you included all possible rectangles with a perimeter of 600 feet.
   The graph will be a smooth curve like the following.



   d. What are the dimensions of the rectangle with the maximum area? What is the area?
   The width and length both equal 150 feet. The area is 22,500 square feet.

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2. Carlos purchased an underground dog fence that includes 500 feet of wire. Carlos also bought an additional 150 feet of wire. He wants to fence in a rectangular region with the maximum area possible using all of the wire.
   a. What dimensions should Carlos use to construct his fence? What is the area of the enclosed region?
      The rectangle with the largest possible area for a given perimeter is a square. The perimeter of the square is 650 feet, so each side measures $162\frac{1}{2}$ feet (650 feet ÷ 4 = 162.5 feet) and the area of the square is $26{,}406\frac{1}{4}$ square feet (162.5 feet x 162.5 feet = 26,406.25 square feet).

   b. How do you know that your answer gives the maximum area for a rectangular region?
      The rectangle with largest possible area for a given perimeter is a square.

3. Consider a set of rectangles that have the same perimeter but different areas. Describe the shape of rectangles with a smaller area versus those with a larger area.
   The longer and thinner the rectangle is, the smaller the area of the rectangle. The closer the rectangle is to a square, the larger the area.

4. What is the smallest possible area for a rectangular yard that has a perimeter of 650 feet?
   This question does not have a numerical answer. A rectangle that measures 10 feet by 315 feet has an area of 3150 square feet. If you make the rectangle half as wide, it will measure 5 feet by 320 feet and has an area of 1600 square feet. Theoretically, a rectangle with a given perimeter can be infinitely long and thin. Realistically, you probably would not want a yard less than 5 feet wide. Many dog runs are between 5 feet and 10 feet wide.

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