Exponential Growth
Use What You've Learned
Answers and Explanations

After watching the video, *Exponential Growth*, complete the following problems.

1. In late 2009, a good CD (Certificate of Deposit) had an "annual yield" of 2%. In other words, an investment would grow exponentially, with a growth rate of 2%. Say you invested $1000 in 2009.

   a. How much is your investment worth in 2014? One way to solve this problem is to think recursively. Recursion involves a repeated application of a procedure to find successive results. How much would you have one year later, in 2010?

      i. What is the initial investment?
      $1000

      ii. How would you find the value of your investment one year later, in 2010?
      There are several methods for solving this problem. Two of those methods are explained below.
      Method 1
      Find 2% of 1000 and add that amount to $1000. The result is $1020.
      $0.02 \bullet 1000 + 1000 = 1020$

      Method 2
      Find 102% of 1000 since the investor receives 100% of the original investment plus 2% of the original investment. The result is $1020.
      $1.02 \bullet 1000 = 1020$

      iii. You can write a recursive equation using two pieces of information. First, you need to know the initial value of the investment. Second, you need to know the repeated procedure, in this case, multiplying by 1.02. Using the terms *Now* and *Next*, write a recursive equation.
      Now $\bullet 1.02 =$ *Next*, starting at $1000.

      iv. Use your recursive equation in (iii) to complete the table and find the value of the CD in six years.

# of years since 2009	0 (2009)	1 (2010)	2 (2011)	3 (2012)	4 (2013)	5 (2014)
Value	$1000	$1020	$1040.40	$1061.21	$1082.43	$1104.08

   b. How much is your investment worth in 2014? Another way to solve this problem is to think explicitly. In other words, if you know the number of years you can use the equation to directly find the value of the CD.
      Using the equation, where t is the number of years since 2009.
      $Value = 1000(1 + 0.02)^t$
      $Value = 1000(1.02)^5$
      $Value = \$1104.08$

c. Which type of equation might be used if you are finding the value of the investment after a couple years? Which type of equation might be used if you are finding the value of the investment after many years?
The answers to these questions are not the same for all students. Many students find the recursive equation quick and easy if only used a few times. If the student has to complete the repeated calculation many times, the explicit equation may be easier.

d. How much would you have in 2029?
Recursively, calculate 102% of the current value to find the next value. The recursive equation is 1.02(Now)=Next where Now is the current value and Next is the next year's value.

This would be continued until 2029.

$1.02 \cdot 1000 = 1020$

$1.02 \cdot 1020 = 1040.40$

$1.02 \cdot 1040.40 = 1061.21$

...

$1.02 \cdot 1428.25 = 1456.81$

$1.02 \cdot 1456.81 = 1485.95$

In twenty years, she would have $1485.95.
You could also find this value by creating an exponential equation. The initial value is 1000 with a growth factor of 1.02 and the exponent of 20. This can be verified by looking at the graph as well.

$1000 \cdot 1.02^{20} = Total$

$1000 \cdot 1.02^{20} = \$1485.95$

2. Assume we want to graph $y = 2^x$. We could use a scale of one unit per inch - the point (2,4) would be 2 inches to the right of the origin, and 4 inches above the origin.

a. How high would the point be that was 6 units to the right of the origin?
Using the equation $y = 2^x$, and substitute 6 for x. $y = 2^6$ or y = 64 inches.

b. How high would the point be that was **one foot** to the right of the origin?
Using the equation $y = 2^x$, and substitute 12 for x. $y = 2^{12}$, or y = 4096 inches, or $341\frac{1}{3}$ feet or 341 feet 4 inches.

c. How high would the point be that was **one yard** to the right of the origin? Give your answer in miles.
Using the equation $y = 2^x$, and substitute 36 for x. $y = 2^{36}$, or y = 68,719,476,736 inches, or 1,084,587.70 miles

3. Assume we have an exponential growth equation $y = a(1+r)^x$, with a growth factor of 0.5 percent.

   a. Fill in the following table:
   The given answers are rounded to four decimal places

x	y
0	162
1	$y = 162(1.005)^1$ or 162.8100
2	$y = 162(1.005)^2$ or 163.6241
3	$y = 162(1.005)^3$ or 164.4422
500	$y = 162(1.005)^{500}$ or 1961.3084

   b. What is the value of *a* in the equation $y = a(1+r)^x$?
   162

4. As of 2009, the population of Wentzville, Missouri was 22,478 people. Since 2000, it has had a population growth rate of 200.88%.

   a. If Wentzville continues to grow at about 200% per decade, what will be the population one decade from now?
   There will be approximately 67,434 people. 22,478 + 200%•22,478 ≈ 67,43

   b. Write an exponential growth equation relating time in decades and total Wentzville population.
   The initial value, or population, in the year 2000 was approximately 7493 and the growth rate is 200% or 2. The equation is *Population* = $7493(1+2)^t$.