Exponential Decay

Use What You've Learned
Answers and Explanations

After watching the video, *Exponential Decay*, complete the following problems.

1. A new Microtosh laptop computer costs $3500 and depreciates with a decay rate of 3% *per month*.

   a. What will the computer be worth in six months? One way to solve this problem is to think recursively.

      i. What is the initial worth of the computer?
      $3500

      ii. How would you find the value of the computer after one month?
      Multiply the initial value $3500 by one minus the decay rate or $3500(1–0.03) or $3500(0.97) = $3395.

      iii. You can write a recursive equation using two pieces of information. First, you need to know the initial value of the computer. Second, you need to know the repeated pattern. In this case, multiplying by 0.97. Using the terms *Now* and *Next*, write a recursive equation.
      Now•0.97 = *Next* , starting at $3500.

      iv. Use your recursive equation in (iii) to complete the table and find the value of the computer in six months.

# of months	0	1	2	3	4	5	6
Value	$3500	$3395	3293.15	3194.35	3098.52	3005.57	2915.40

   b. What will the computer be worth in six months? Another way to solve this problem is to think explicitly. In other words, if you know the number of months you can use the equation to directly find the value of the computer.
   Using the equation where t is the number of months since the purchase.

   $Value = 3500(1-0.03)^t$

   $Value = 3500(0.97)^6$

   $Value = \$2915.40$

   c. Which type of equation might be used if you are finding the value of the computer after a couple months? Which type of equation might be used if you are finding the value of the computer after many years?

The answers to these questions are not the same for all students. Many students find the recursive equation quick and easy if only used a few times. If the student has to complete the repeated calculation many times, the explicit equation may be easier.

d. What is the computer worth after three years?

$Value = 3500(1-0.03)^t$

$Value = 3500(0.97)^{36}$

$Value = \$1169.10$

e. How do the explicit equation and recursive equation relate?
There are many possible answers but a student should note that the initial value and the repeated pattern are needed for both. The initial value in the recursive equation is the first "Now" value. In the explicit equation it is the value multiplied by decay factor raised to a power. 0.97 is the decay factor in the explicit equation. 0.97 is multiplied by the current value to find the next value in the recursive equation.

f. What is the decay rate *per year*?
$1-0.97^{12} = 30.6\%$

2. Assume we have an exponential relationship with a decay rate of 15 percent.
   a. Fill in the following table:

x	y
0	42,264
1	35,924.40; $y = 42264(1-0.15)^1$ or $42,264 \bullet 0.85 = 35,924.40$
2	30,535.74; $y = 42264(1-0.15)^2$ or $35,924.40 \bullet 0.85 = 30,535.74$
3	25,955.38; $y = 42264(1-0.15)^3$ or $30,535.74 \bullet 0.85 = 25,955.38$
100	0.00369712; $y = 42264(1-0.15)^{100}$

b. For which values of x is it easy to find y recursively? For which values of x would it be tedious to find y recursively?
Finding y recursively for x = 1, 2, and 3 is reasonable. Finding y for x = 100 recursively would take a long time. For this, a student should use the explicit equation.

c. What is the value of *a* in the equation $y = a(1-r)^x$?
42,264 since this is the value of y when x = 0.

3. Every living thing contains a certain amount of a radioactive element called carbon-14. As soon as something dies, the carbon-14 gradually goes away. Its decay factor is approximately 0.9998790392, assuming time is measured in years.

   a. Assume we had 10 grams of carbon-14. How much would we have in 100 years?

      $A = 10(0.9998790392)^t$

      $A = 10(0.9998790392)^{100}$

      $A = 9.8798$ grams

   b. Assume an organism had C_0 grams when it died. How many grams would it have in 5730 years?

      $A = C_0(0.9998790392)^{100}$

      $A = C_0(0.5)$

      or

      $A = 0.5C_0$ or half of the original

      5730 years is called the half-life.

   c. We examine a bone that a merchant tells us is 2,000 years old. We determine that it has 90% of its original carbon-14. Is the bone older, younger, or about 2,000 years old?

      If it were 2000 years old we can calculate the percent of original carbon using $(0.9998790392)^{2000}*100 = 78.51\%$. So the bone is much younger.

4. We pass a gallon of sludgy water through a filter to purify it. Each inch of filter removes 30% of the contaminants. Assume that we start with 2000 grams of contaminant in the water.

   a. Write a recursive equation to find the grams of contaminant for each inch of filter.

      $Next = (1 - 0.3) \cdot Now$

      $Next = 0.7Now$ starting at 2000 grams

   b. Find the number of contaminants in the water if the filter was one inch.

      $Next = 0.7Now$

      $Next = 0.7 \cdot 2000$

      $Next = 1400$ grams

c. Write the explicit equation that gives the amount of contaminant remaining in the water after it has passed through *x* inches of filter

$C = 2000(1-0.3)^x$ or $C = 2000(0.7)^x$ where x is inches of the filter and C is grams of contaminant.

d. Assume the water is safe to drink when there is less than 30 grams of contaminant in it. If we pass it through a one-foot long filter, is it drinkable?

$C = 2000(1-0.3)^{12}$

$C = 27.68$ grams

It is drinkable since there are fewer grams than 30.