



You have a credit card with an *Annual Percentage Rate* (APR) of 21.15%. At the beginning of the year you have a balance of \$200. Say you do not make any additional purchases and do not pay off the current balance. The interest is compounded monthly, and you are not charged any late fees.

1. How much would you owe after one month?
 - a. What is the initial balance?
\$200
 - b. What is the monthly interest rate for the given APR? Note that 21.15% is the annual rate.
 $21.15\% \div 12 = 1.7625\%$
 - c. What is the balance you owe after one month?

There are several methods for solving this problem. Two of those methods are explained below.

Method 1: Find 1.7625% of \$200 and add that amount to \$200. The result rounded to the nearest cent is \$203.53. ($0.017625 \cdot 200 + 200 = \203.525)

Method 2: Find 101.7625% of \$200 since the amount owed is 100% of the balance plus 1.7625% of the balance. The result rounded to the nearest cent is \$203.53. ($1.017625 \cdot 200 = \203.525)

2. How much would you owe after one year?
 - a. One way to solve this is to think recursively. Recursion involves a repeated application of a procedure to find successive results. You can write a recursive equation using two pieces of information. First, you need to know the initial amount you owe. Second, you need to know the repeated procedure. In this case, multiplying by 1.017625 to find the amount owed at the end of the month. Using the terms *Now* and *Next*, write a recursive equation.
 $Now \cdot 1.017625 = Next$, starting at \$200
 - b. Use your recursive equation to complete the table and find the amount you would owe after one year. Note the values in the table are rounded to the nearest cent, but the calculations were determined without rounding.

Month	NOW (Balance at the Beginning of the Month)	NEXT (Balance at the End of the Month)
1	\$200	\$203.53
2	\$203.53	\$207.11
3	\$207.11	\$210.76
4	\$210.76	\$214.48
5	\$214.48	\$218.26
6	\$218.26	\$222.10
7	\$222.10	\$226.02
8	\$226.02	\$230.00



9	\$230.00	\$234.06
10	\$234.06	\$238.18
11	\$238.18	\$242.38
12	\$242.38	\$246.65

- c. Another way to solve this problem is to think explicitly. In other words, if you know the number of months you can use an equation to directly find the amount you owe after one year. The equation for the amount owed after n months in this situation is $amount\ owed = P(1 + r)^n$ where P is the initial balance, r is the monthly interest rate, and n is the number of months. Using this equation, how much would you owe at the end of one year?

$$\text{Amount Owed} = 200(1 + 0.017625)^{12}$$

$$\text{Amount Owed (rounded to the nearest cent)} = \$246.65$$

- d. Compare this equation to your table.

Note: If you rounded the values in the table before multiplying, the total amount owed after one year may not exactly match the amount determined with the equation.

Explain why this equation works.

You multiply the balance by the quantity 1 plus the interest rate in order to determine the total amount owed. If you did not add one to the interest rate, the result would only give you the amount of interest owed. In this case, you multiply the previous month's ending balance by 1.017625 to find the total amount owed at the end of the next month. As a result you repeatedly multiply the beginning balance by 1.017625 (see table below). You can represent this repetition by using exponents.

Month	NOW (Balance at the Beginning of the Month)	NEXT (Balance at the End of the Month)
1	\$200	$200(1.017625)$
2	$200(1.017625)$	$200(1.017625)(1.017625)$
3	$200(1.017625)(1.017625)$	$200(1.017625)(1.017625)(1.017625)$
4	$200(1.017625)(1.017625)(1.017625)$	$200(1.017625)^4$
5	$200(1.017625)^4$	$200(1.017625)^5$
6	$200(1.017625)^5$	$200(1.017625)^6$
7	Etc.	$200(1.017625)^7$
8		$200(1.017625)^8$
9		$200(1.017625)^9$
10		$200(1.017625)^{10}$
11		$200(1.017625)^{11}$
12		$200(1.017625)^{12}$

3. What is the *Effective Annual Rate* (EAR) for this credit card? Note that the EAR takes into account that interest is compounded 12 times per year.
You start with a balance of \$200 and pay a total of \$246.65, so you pay \$46.65 in interest. Divide 46.65 by 200 to find the percent of interest paid. ($46.65 \div 200 = .23325$ or 23.325%)
4. How much would you owe after one year if you assume the APR is a simple interest rate, that is, compounding occurs only one time?
There are several methods for solving this problem. Two of those methods are explained below.
Method 1: Find 21.15% of \$200 and add that amount to \$200. The result rounded to the nearest cent is \$242.30. ($0.2115 \cdot 200 + 200 = \242.30)
Method 2: Find 121.15% of \$200 since the amount owed is 100% of the balance plus 21.15% of the balance. The result rounded to the nearest cent is \$242.30. ($1.2115 \cdot 200 = \242.30)
5. What is the difference between the APR and EAR for this scenario?
 $23.325\% - 21.15\% = 2.175\%$
6. Many credit card statements only list the APR. How might this mislead a customer?
The interest rate is actually a little higher when you consider compounding from month to month.
7. Which rate, the APR or EAR, is more useful to you when thinking about credit card debt?
Both numbers are useful. The APR is needed and more useful if you want to calculate the monthly balance in any future month. The EAR is more useful to determine actual costs on a yearly basis.
8. What other factors will impact the total amount owed on a credit card if you do not make any payments for one year?
Credit card companies will charge a late fee each month. On the video the late fee is \$39.00. Now there is a law that caps the late fee at \$25.00 per month. This amount will be added to your balance and you will start paying interest on the late fee. This will significantly impact the total amount you owe at the end of one year. Late fees may also impact your credit history negatively.