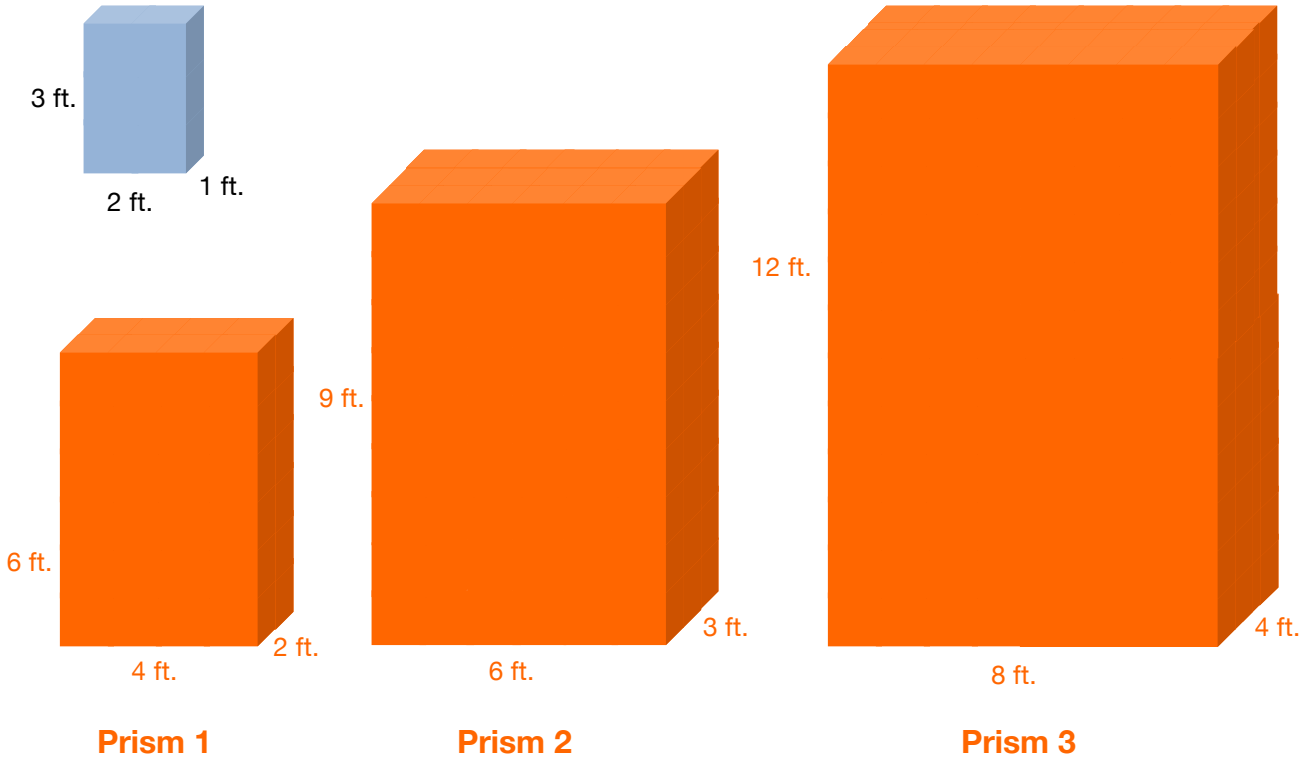
Changing Dimensions: Volume

Use What You've Learned
Answers and Explanations

1. Sketch three different rectangular prisms that are similar to the one shown below using the following directions:
   Prism 1: Double the length, width, and height (scale factor = 2).
   Prism 2: Triple the length, width, and height (scale factor = 3).
   Prism 3: Quadruple the length, width, and height (scale factor = 4).



2. Find the volume of each of the rectangular prisms.

Original Prism	Prism 1	Prism 2	Prism 3
V = l • w • h = 2 • 1 • 3 = 6 cu. ft.	V = l • w • h = 4 • 2 • 6 = 48 cu. ft.	V = l • w • h = 6 • 3 • 9 = 162 cu. ft.	V = l • w • h = 8 • 4 • 12 = 384 cu. ft.

3. Study your results for problems 1 and 2. What happens to the volume of a rectangular prism when you double the dimensions, triple the dimensions, quadruple the dimensions, or make the dimensions n times as large (scale factor = n)? Record your answers in the following table.

Scale Factor	Volume
1	V
2	8V
3	27V
4	64V
n	$(n^3)V$

The volume of the original prism is 6 cubic feet. The volume of prism 1 is 48 cubic feet, which is 8 times as large as the original prism (8 x 6 = 48). The volume of prism 2 is 162 cubic feet, which is 27 times as large as the original prism (27 x 6 = 162). The volume of prism 3 is 384 cubic feet, which is 64 times as large as the original prism (64 x 6 = 384). The volume factor is the scale factor cubed.

4. Sketch a rectangular prism that is similar to the following prism. Make the length, width, and height one half of the original length, width, and height. What happens to the volume of a prism when you make the dimensions ½ as large? Does this follow the pattern you described in problem 3?

$V = l \cdot w \cdot h$
$= 4 \cdot 2 \cdot 1$
$= 8$ cu. m

$V = l \cdot w \cdot h$
$= 2 \cdot 1 \cdot ½$
$= 1$ cu. m

The volume of the smaller prism is $\frac{1}{8}$ the volume of the original prism. Yes, this follows the pattern shown in problem 3. The scale factor is $\frac{1}{2}$ and the volume factor is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$, or $\frac{1}{8}$. The volume factor is the scale factor cubed.

5. A truck holds 8 large boxes. If a smaller box is similar in shape, but all the dimensions are one half the size of the large box, how many boxes can the truck hold?

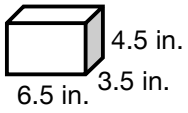
   If the dimensions of the small box are $\frac{1}{2}$ the dimensions of the large box, the volume of the small box is $\frac{1}{8}$ the volume of the large box. That means eight small boxes will fit in each large box. If the truck holds 8 large boxes, it will hold 8 times 8, or 64 small boxes.

6. A DVD case is 5.5 inches by 0.5 inches by 7.5 inches. How many DVD cases will fit on a shelf that measures 22 inches by 30 inches by 2 inches?

   30 in. ÷ 7.5 in. = 4 22 in. ÷ 5.5 in. = 4 2 in. ÷ 0.5 in. = 4

   Four DVD cases will fit along each dimension of the shelf. That means 4 x 4 x 4, or 64 DVD cases will fit on the shelf.

7. One section of a freezer case at a grocery store is tightly packed with 27 containers of ice cream. The dimensions of one container of ice cream are shown below. What are the dimensions of that section of the freezer case? Assume that each dimension is greater than 6.5 inches.



   If each dimension is greater than 6.5 inches, and the containers fit tightly, there must be more than one container of ice cream placed along each dimension of the shelf. Twenty-seven containers of ice cream can placed in a 3-by-3-by-3 rectangular prism as shown below. The dimensions of the prism are 19.5 inches (3 x 6.5) by 10.5 inches (3 x 3.5) by 13.5 inches (3 x 4.5).