Standard Deviation
Extend Your Learning
Answers and Explanations

After watching the video, *Standard Deviation*, complete the following problems.

1. My sister and I do sit-ups each morning. She does them more consistently and has done them for a much longer time. Here are her records:

Number of sit-ups	Number of days I did that many
121	27
122	75
123	143
124	200
125	250
126	195
127	132
128	62
129	19

   a. For how many days has she been keeping records of her sit-ups?
   To find this solution add all values in the "Number of days" column. 1103 days

   b. What is the mean number of sit-ups she has done per day?
   Adding all the sit-ups totals 137,777 and dividing by 1103 days gives 124.911 sit-ups per day or about 125 sit-ups per day.

   c. What is the standard deviation? Hint: There were 27 days when there were 121 sit-ups. 1.770

Number of sit-ups	$x - \bar{X}$	$(x - \bar{X})^2$	$(x - \bar{X})^2$ •# of days
121	121-125	$(-4)^2=16$	16 • 27 = 432
122	122-125	$(-3)^2=9$	9 • 75 = 675
123	123-125	$(-2)^2=4$	4 • 143 = 572
124	124-125	$(-1)^2=1$	1• 200 = 200
125	125-125	$(0)^2=0$	0 • 250 = 0
126	126-125	$(1)^2=1$	1 • 195 = 195
127	127-125	$(2)^2=4$	4 • 132 = 528
128	128-125	$(3)^2=9$	9 • 62 = 558
129	129-125	$(4)^2=16$	16 • 19 = 304
		Total:	3464

$$\sqrt{\frac{3464}{1103}} \approx 1.77$$

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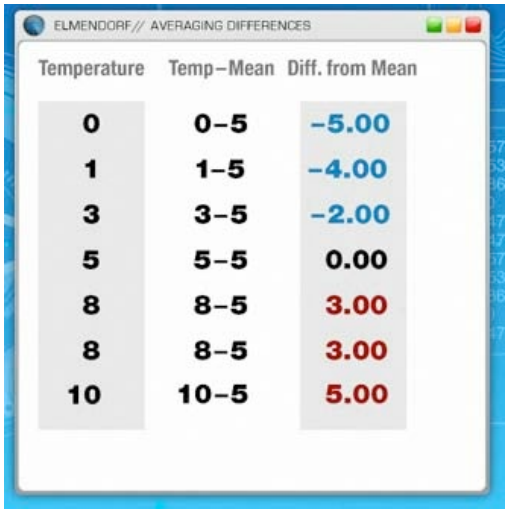
2. In the video, Emily experimented with trying to measure the spread of data by averaging the difference between each data point and the mean. When she did so, she got a mean of zero every time. In other words, her tables all looked like this:



Temperature	Temp–Mean	Diff. from Mean
0	0–5	–5.00
1	1–5	–4.00
3	3–5	–2.00
5	5–5	0.00
8	8–5	3.00
8	8–5	3.00
10	10–5	5.00

Notice that the sum of the differences from the mean is zero, making the average difference from the mean zero as well. This is why we had to do something different to find the standard deviation – squaring the differences and taking the square root at the end.

Figure out why this sum is always zero, and explain your discovery. To simplify things, you can assume we have seven data points if you like, but it is not necessary.

Answers can be written out in words, but it is actually easiest to use algebra.

The mean, μ, is given by $\mu = \frac{x_1 + x_2 + \ldots + x_7}{7}$. Now the differences are as follows:

$$x_1 - \frac{x_1 + x_2 + \ldots + x_7}{7}$$

$$x_2 - \frac{x_1 + x_2 + \ldots + x_7}{7}$$

M

$$x_7 - \frac{x_1 + x_2 + \ldots + x_7}{7}$$

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Adding these gives:

$$x_1 + x_2 + \ldots + x_7 - 7\left(\frac{x_1 + x_2 + \ldots + x_7}{7}\right)$$
$$= x_1 + x_2 + \ldots + x_7 - \left(x_1 + x_2 + \ldots + x_7\right)$$
$$= 0$$

3. Fill a small bowl with unpopped popcorn and a small bowl with popped popcorn. [You can substitute anything that is two different sizes for this problem.] Then grab a handful of unpopped popcorn, count the kernels, and return them to the bowl. Do this seven times, recording the number of kernels each time. Do the same thing with the popped popcorn.

   a. Which set of data do you predict will have a larger standard deviation? Why?
   Most likely the number of kernels will have a larger standard deviation because there will be a greater variance in the number of kernels grabbed for the smaller item than the larger item.

   b. Find the standard deviation for both sets of data and compare with your predictions.
   Answers will depend on each set of data.

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