Introduction to Probability

Extend Your Learning
Answers and Explanations

Many board games use two dice, and the sum of the numbers on the dice determines how far a player moves. Consider a game that uses two dice but the difference between the two numbers on the dice determines how far a player moves.

1. The possible differences between the numbers on two ordinary dice are 0, 1, 2, 3, 4, and 5. (Ordinary dice have six faces with dots representing 1 through 6.) Explain why these are the only possible differences.
   Possible differences range from 0 to 5 because the only numbers represented on the dice are 1, 2, 3, 4, 5, and 6. Since the largest possible number is 6 and the smallest is 1, the largest possible difference is 5. If you roll doubles, the difference is 0. If you roll any other combination of numbers, the difference is greater than 0 and less than 5.

2. Predict which difference is most likely and which is least likely. Determine your prediction without listing all the ways to roll each difference. Explain your reasoning.
   A difference of 5 is probably least likely because you must roll a 1 and a 6 to get a difference of 5. It appears that the smaller the difference the more ways there are to roll a number combination equaling that difference.

3. Roll two dice 36 times. After each roll, determine the difference between the numbers and record a tally mark below the difference. After 36 rolls, count the tally marks and write the total number of times each difference occurred.

Difference	0	1	2	3	4	5
Tally Marks			Answers	will vary.		
Total						

4. Based on your experiment, what is the probability of rolling each difference?

P (0) = $\frac{\square}{36}$

P (1) = $\frac{\square}{36}$

P (2) = $\frac{\square}{36}$

P (3) = $\frac{\square}{36}$

P (4) = $\frac{\square}{36}$

P (5) = $\frac{\square}{36}$

Answers will vary, but you can represent each probability as a fraction. The numerator is the number of times you rolled each difference and the denominator is the total number of times you rolled the two dice, or 36. The sum of the numerators will be 36, so that the sum of the probabilities of all possible outcomes equals one.

5. In order to determine the theoretical probability of rolling each difference, list all the ways you could roll each difference in the table shown below. As you complete the table, think about rolling one red die and one white die. For example, you could roll a 1 on the red die and a 3 on the white die, or you could roll a 3 on the red die and a 1 on the white die.

0	1	2	3	4	5
1 and 1	1 and 2	1 and 3	1 and 4	1 and 5	1 and 6
2 and 2	2 and 1	3 and 1	4 and 1	5 and 1	6 and 1
3 and 3	2 and 3	2 and 4	2 and 5	2 and 6	
4 and 4	3 and 2	4 and 2	5 and 2	6 and 2	
5 and 5	3 and 4	3 and 5	3 and 6		
6 and 6	4 and 3	5 and 3	6 and 3		
	4 and 5	4 and 6			
	5 and 4	6 and 4			
	5 and 6				
	6 and 5				

6. Describe any patterns you see in your table. Do the patterns hold for all differences? Why or why not?
The following list includes some patterns. You may have found additional patterns.
   - The number of ways of rolling each difference is a multiple of 2.
   - There are two ways to get each number combination, except for the doubles. For example, you can roll a 1 on the red die and a 6 on the white die, or you can roll a 6 on the red die and a 1 on the white die.
   - The smaller the difference, the greater number ways to roll that difference, except for a difference of 0. This is because there is only one way to roll each double.
   - There are 2 ways to roll a difference of five, 4 ways to roll a difference of four, 6 ways to roll a difference of three, 8 ways to roll a difference of two, 10 ways to roll a difference of one, and 6 ways to roll a difference of zero. Starting with a difference of one, the number of ways to roll each difference decreases by 2.

7. What is the theoretical probability of rolling each difference?

$P(0) = \frac{6}{36}$ $P(3) = \frac{6}{36}$

$P(1) = \frac{10}{36}$ $P(4) = \frac{4}{36}$

$P(2) = \frac{8}{36}$ $P(5) = \frac{2}{36}$

8. Are your experimental probabilities from problem 4 the same as the theoretical probabilities? Why or why not?
   Your answer is probably "no." As with any experiment, your results will vary from the theoretical probabilities.

9. How do you think your experimental probabilities would change if you rolled the dice 1000 times rather than 36 times?
   If you roll two dice a large number of times, like 1000, you can expect your results to be close to the theoretical probability. Generally, the larger the number of times you roll the dice, the closer the experimental probability will be to the theoretical probability.