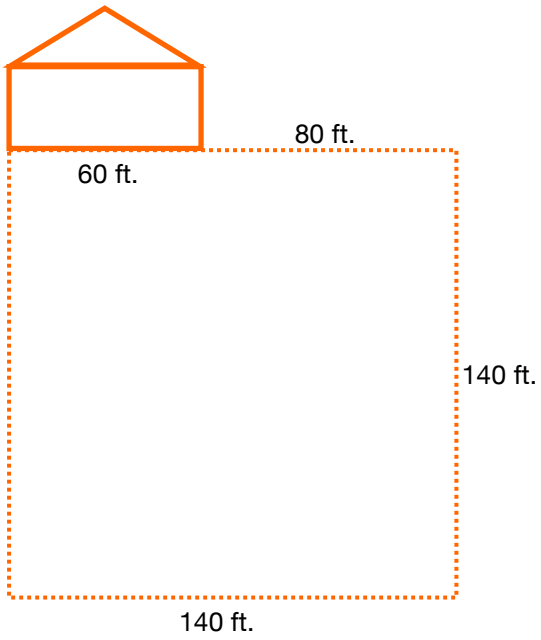
Fixed Perimeter, Changing Area
Extend Your Learning
Answers and Explanations

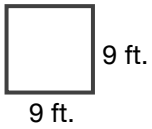
1. After observing how well the underground fence works, Al's neighbor, Rod, decides to put in an underground dog fence for his dog. Rod buys a kit containing 500 feet of wire. He also wants his fence to be in the shape of a rectangle and enclose the maximum area. The only difference is that Rod wants to use the backside of his house as part of the perimeter. Rod's house is 60 feet wide.
   a. What dimensions should Rod use to construct his fence? What will be the area of the enclosed region?
   Since Rod wants to use the backside of his house as part of the perimeter, the perimeter of the yard will be 560 feet, rather than 500 feet. The rectangle with the largest area for a fixed perimeter is a square. A square with a perimeter of 560 feet, measures 140 feet on each side (560 feet ÷ 4 = 140 feet). The dimensions of the yard will be 140 feet by 140 feet and will have an area of 19,600 square feet (140 feet x 140 feet = 19600 square feet).

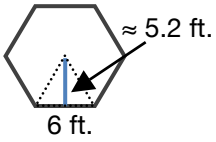


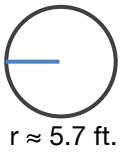
   Note: The house shown in the diagram can be positioned anywhere along the perimeter of the yard.

   b. How do you know that you have determined the largest possible area?
   Given a fixed perimeter, the rectangle with the largest area is a square.

2. Zaid wants to purchase a trampoline. He is comparing a square trampoline, a hexagonal trampoline, and a round trampoline. All of the trampolines have a perimeter of 36 feet. The jumping area of each trampoline is shown below.







a. Predict which trampoline has the largest jumping area.
If you base your prediction on appearance, any of the figures shown above may look the largest.

b. Determine the jumping area of each trampoline. (Hint: The formula for area of a triangle is $A = \frac{1}{2}bh$ and the formula for area of a circle is $A = \pi r^2$.) Was your prediction correct?

Square

$A = bh$

$= 9 \cdot 9$

$= 81$ square feet

Hexagon

$A = 6 (\frac{1}{2}bh)$

$\approx 6 (\frac{1}{2} \cdot 6 \cdot 5.2)$

$\approx 6 (15.6)$

≈ 93.6 square feet

Circle

$A = \pi r^2$

$\approx \pi \cdot 5.7^2$

$\approx \pi \cdot 32.49$

≈ 102.02 square feet

Note: One way to find the area of the hexagonal region is to divide it into six congruent triangles, determine the area of one triangle, and multiply the area times six.

3. What shape do you think has the largest area for a fixed perimeter? Explain your reasoning.
If you only consider rectangles, the rectangle with the largest area (given a fixed perimeter) is a square. If you consider all possible shapes, the above solution shows a circle has a larger area than a square with the same perimeter. Mathematicians have proven that the shape with the largest area for a given perimeter is a circle.