Exponential Growth
Extend Your Learning
Answers and Explanations

After watching the video, *Exponential Growth*, complete the following problems.

1. Assume I gave you one penny on July 1st, two pennies on July 2nd, four pennies on July 3rd, etc., doubling every day.

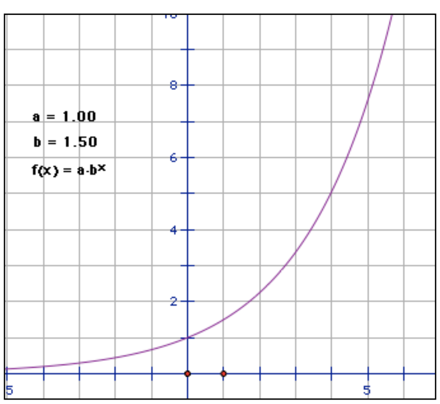
   a. How much money would I give you on August 1st?
   There are 31 days in July, so the number of pennies given doubles 31 times, or 2^{31}.
   This is 2,147,483,648 pennies or $21,474,836.48.

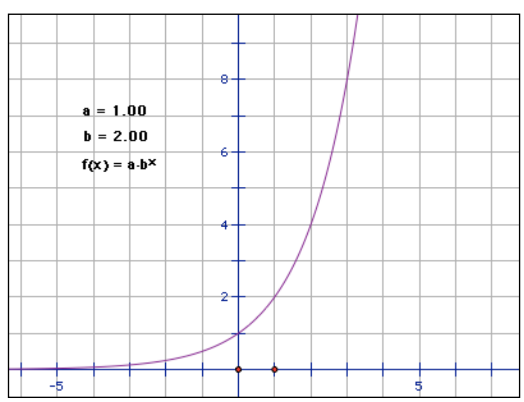
   b. If we wrote the equation, $y = a\left(1+r\right)^{x}$, where x =1 on July 1st, x = 2 on July 2nd, and so forth, find the values of *a* and *r*.
   a = 0.5 and *r* = 1, or 100%
   The growth rate is represented by "r" and is 1 or 100%.
   The value of the dependent variable when n equals 0 is represented by "a." In our equation, this value is 0.5 since $0.5(1 + 1)^0 = 0.5$. It is half of 1.

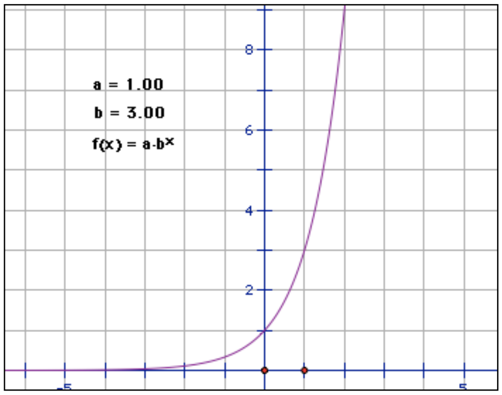
   c. What is the growth rate?
   100%. Recall the growth factor is 1+ r and the growth rate is r. In this problem the growth factor is 2 and the growth rate is 1, or 100%.

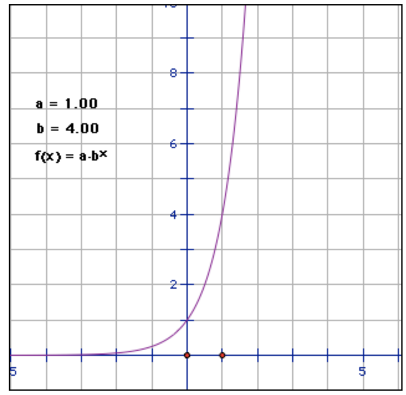
2. The video used the equation $y = a\left(1+r\right)^{x}$. An equivalent equation, $y = ab^{x}$, where $b = 1+r$ is frequently used as well.

   a. For $a = 1$ and $b > 1$, predict what will happen to a graph when the value of b changes. Try at least four different values for b.
   As *b* increases the value of the function increases at an increasing rate. Informal ways of describing this change might be "gets bigger faster" or "goes up quicker". Examples given were created using Geometer's Sketchpad®.

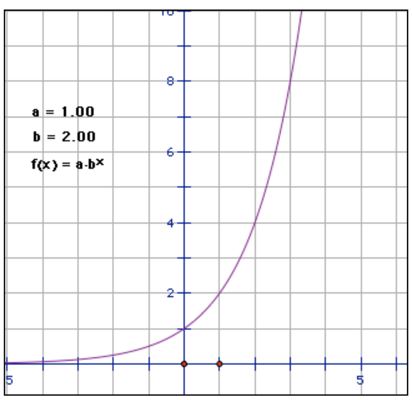


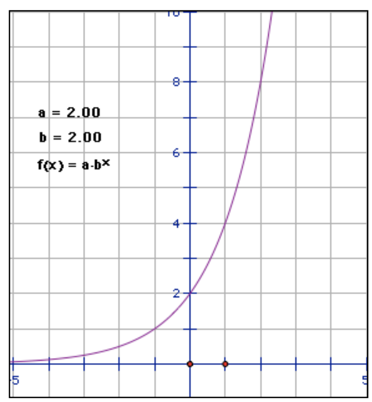


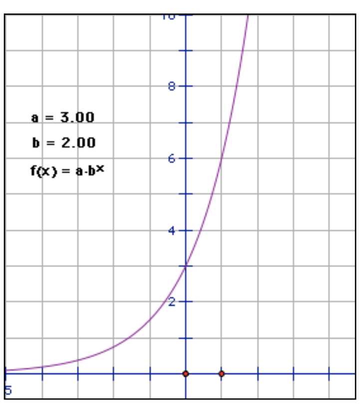


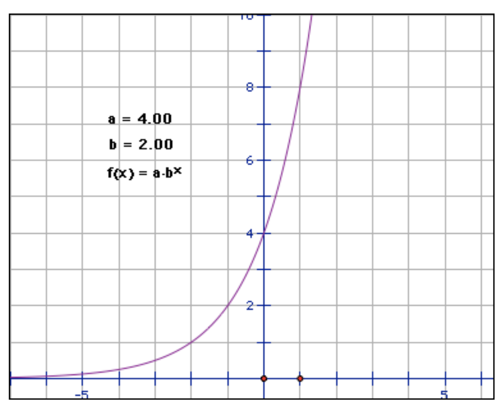


b. For $a > 1$ and $b = 2$, predict what will happen to a graph when the value of a changes. Try at least four different values for a.
As *a* increases, the shape of the graph does not change but the y-intercept does change. The y-intercept for an exponential function is at (0, *a*). Informal ways of describing the change in the graph as *a* increases might be: “the graph moves up” or “the graph has a vertical shift.” Examples given were created using Geometer’s Sketchpad®.

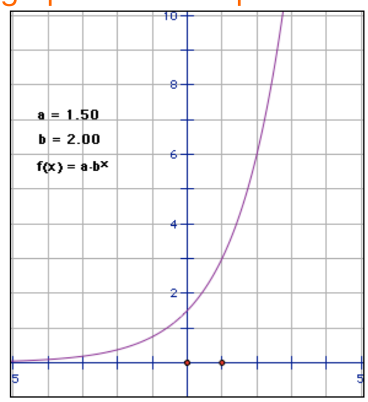


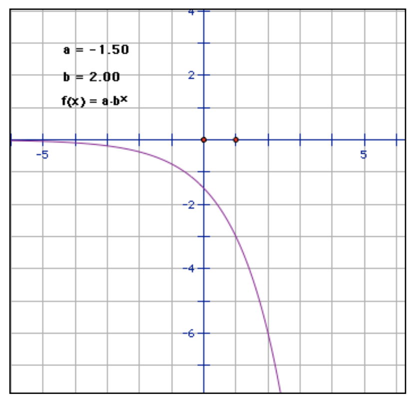






c. Let $b = 2$. Predict what will happen to a graph when the value of a is negative. Try several different values for a.
When a is negative, the graph is a reflection about the x-axis of the corresponding graph when a is positive.





d. Let $a = 1$. Predict what will happen to a graph when the value of b is negative. Try several different values for b.
An exponential equation, by definition, can only have a positive value of b. If students use technology to graph any of these equations, they will encounter an error.

3. In 1985, there were 15,948 diagnosed cases of AIDS in the United States. In 1990 there were 156,024. Scientists said that if there was no research done, the disease would grow exponentially.

a. Compute the number of cases this model predicted for the year 2000.
15,948 is the initial value and the growth rate is found by dividing 156,024 by 15,948. This gives the growth over the five years, so this value is taken to the one-fifth power.
$\left(\frac{156024}{15948}\right)^{\frac{1}{5}} \approx 1.578$ So, the general equation is $y=15948*(1+0.578)^t$ where t is the number of years since 1985 and y is the number of AIDS cases.
In the year 2000, t=15. Using the general equation, $14,938,645 = 15,948(1.578)^{15}$.
The model predicted 14,938,645 cases of AIDS in the United States in the year 2000.

b. By the year 2000, there were 774,467 cases of AIDS. Was the prediction accurate? Discuss possible flaws in the model.
The model is not accurate. Between 1985 and 2000, more was learned about the disease, including methods of prevention. The result could have been a slowing in the growth rate. Answers may vary.