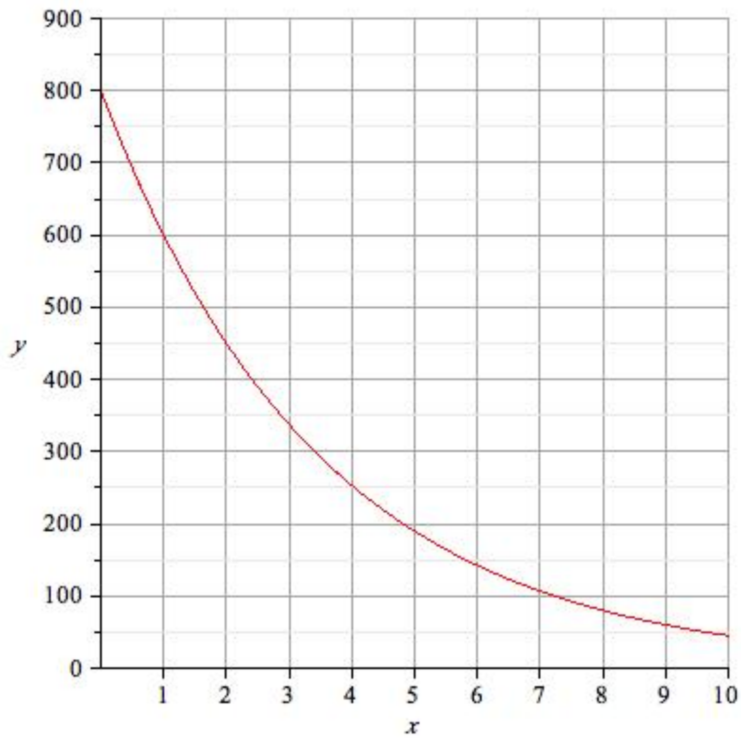
Exponential Decay
Extend Your Learning
Answers and Explanations

After watching the video, *Exponential Decay*, complete the following problems.

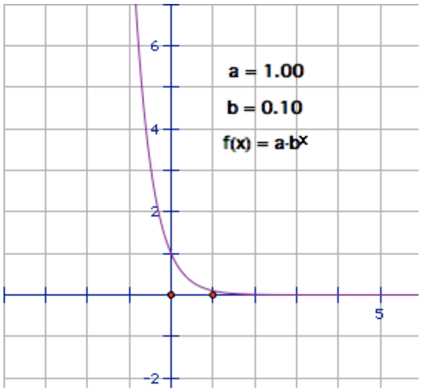
1. The following is a graph of $y = a(1-r)^x$. Find a, r (the decay rate) and the decay factor.

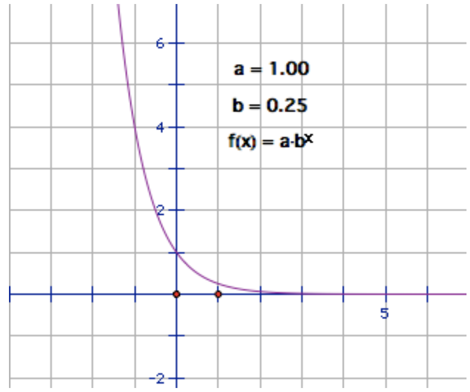


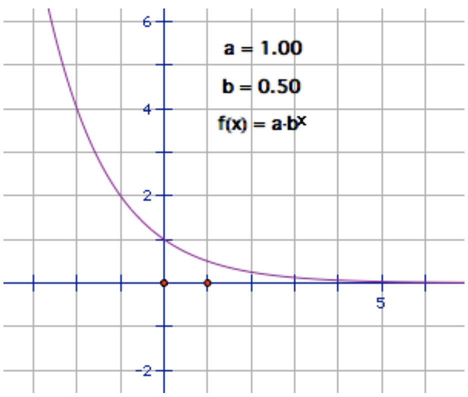
Since the value of y=800 when x=0 then a=800. Looking at the rate of change when x=1, y=600. This means when x changed by 1 the decay factor was $\frac{600}{800} = 0.75$ or 75%. This means the decay rate is 25% or r = 0.25.

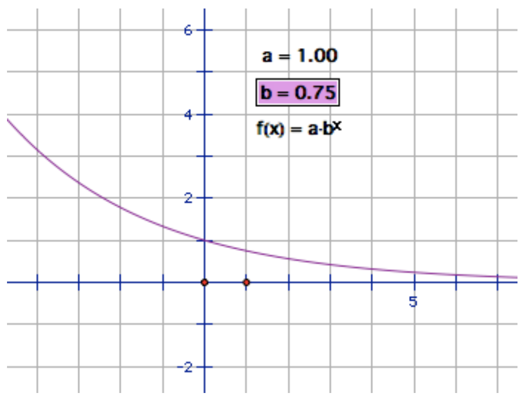
2. The video used the equation $y = a(1-r)^x$. An equivalent equation $y = ab^x$ where $b = 1-r$ is frequently used as well. Use a graphing calculator or computer to investigate changes in the graph.

   a. Let $a = 1$ and $0 < b < 1$ and predict what will happen to a graph when the value of b changes. Try at least four different values for b.
   As b increases, the value of the function decreases at a decreasing rate. An informal ways of describing this change might be: “gets smaller slower”. Examples given were created using Geometer’s Sketchpad®.



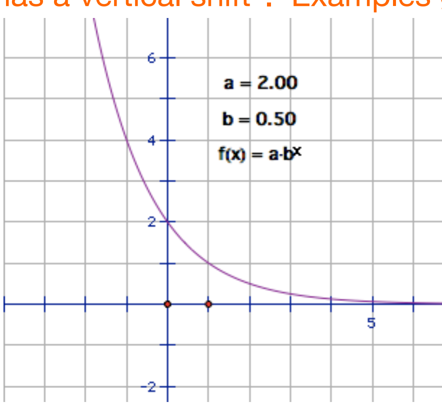


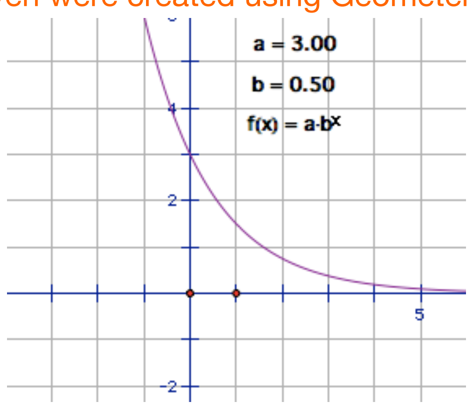


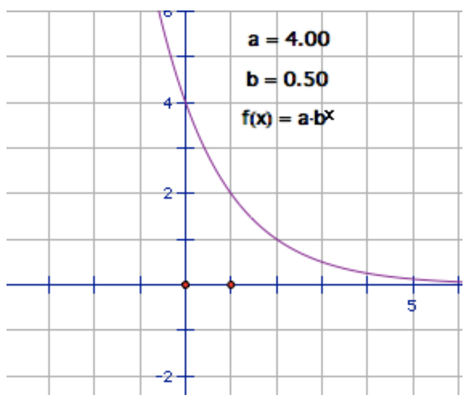


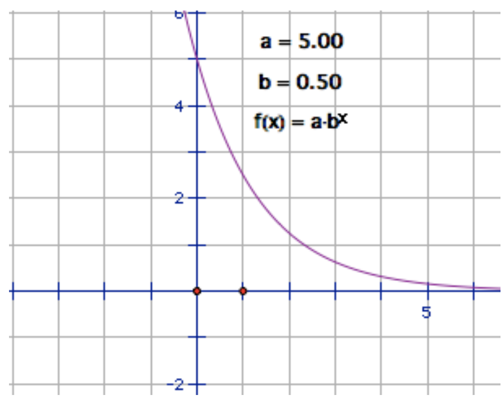
b. Let $a > 1$ and $b = 0.5$ and predict what will happen to a graph when the value of a changes. Try at least four different values for a.

As *a* increases the shape of the graph does not change but the y-intercept does change. The y-intercept for an exponential function is at (0, *a*). Informal ways of describing the change as *a* increases might be: "the graph moves up" or "the graph has a vertical shift". Examples given were created using Geometer's Sketchpad®.

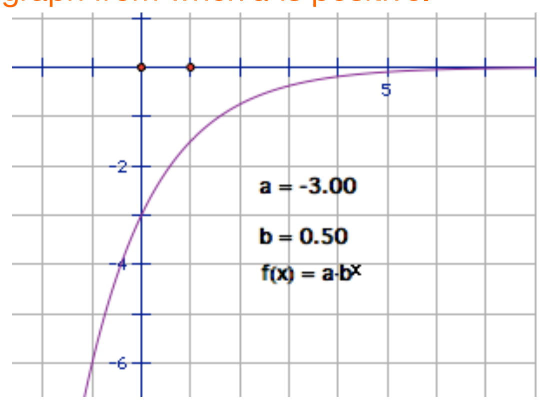




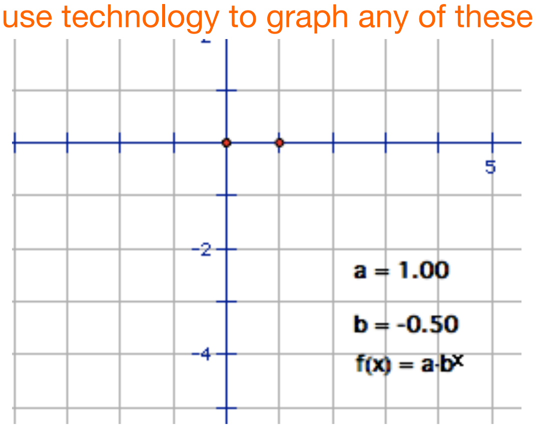




c. Let $b = 0.5$ and predict what will happen to a graph when the value of a is negative. Try several different values for a.
When *a* is negative, the graph is a reflection about the x-axis from the corresponding graph from when *a* is positive.



d. Let $a = 1$ and predict what will happen to a graph when the value of b is negative. Try several different values for b.
An exponential equation by definition can only have a positive value of b. If students use technology to graph any of these equations, they will encounter an error.



3. When we learn a list of data, such as vocabulary words in another language, it is theorized that our memory decays exponentially over time unless we continually practice. Assume Mia learned 100 words and knew them all when it was time to be tested. Further, assume that she only knew 95 words one month later.

   a. How many words did she know one year later?

   The decay factor for one month is $\frac{95}{100} = 0.95$. She retains 95% of the words each month. The initial value is 100 words.
   Recursive equation: $Next = 0.95 \cdot Now$ starting at 100
   Explicit equation: $N = 100(0.95)^t$ where t is time in months and N is the number of words retained.
   $N = 100(.95)^{12}$
   $N = 54$ words

   b. How many words did she know two years later?
   $N = 100(.95)^{24}$
   $N = 29$ words

   c. How long did it take before she remembered only one word?
   89 years gives 1.04 words. But you can argue that 82 years gives 1.49, which is closer to 1 than it is to 2. Or you can say that 77 years gives 1.93, which is less than 2. This is an opportunity to discuss that mathematical models may be reasonable only for a distinct period of time. One word may be beyond the practical range.