Volume of Prisms and Cylinders

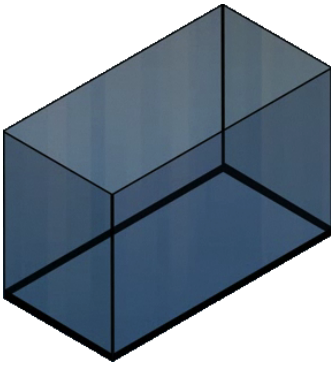
A Closer Look at the Video

After watching the *Volume of Prisms and Cylinders* video, make sense of the mathematics by reading through the problem situation and solution. Use the comments and questions in bold to help you understand how to determine the volume of a prism or cylinder.

Problem: *Kali is at Gill's Pet Shop checking out aquariums. She has selected one fish, Bubbles, and is now looking for more fish and a new aquarium. The fish tanks come in several different shapes and sizes. The experts at the pet store said the number of fish Kali can have depends on the volume of the tank that she chooses. Help Kali determine the volume of different aquariums so she can choose the right one for her fish.*

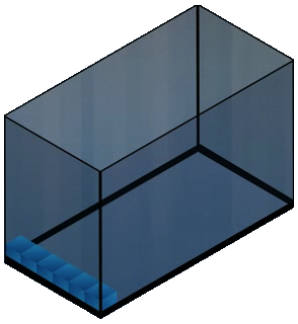
What is volume?
Volume is the amount of space that a three-dimensional shape, like a fish tank, contains. One way to measure volume is to determine the number of cubes it takes to fill a three-dimensional shape.

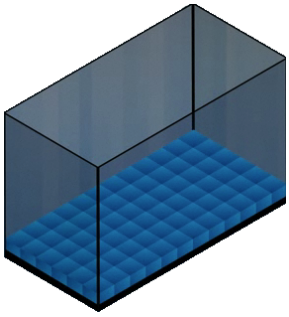
Let's start with this aquarium, which is a rectangular prism. How can we figure out how many cubes it will take to fill the aquarium?



We cannot just pour a bunch of cubes in. We'll need to stack the cubes so that they fill the aquarium without any gaps. The dimensions of each cube are 1 inch by 1 inch by 1 inch, so the volume of one cube is 1 cubic inch. The total number of cubes that will fit in the aquarium equals the volume of the aquarium in cubic inches.

First determine the number of cubes that will cover the bottom of the aquarium. Each row contains 6 cubes, and there are a total of 12 rows.





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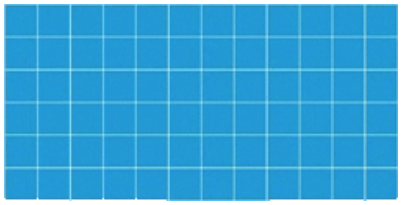
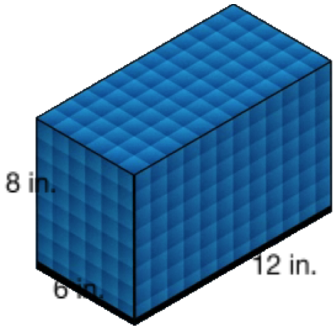
What is the area of the base of the rectangular prism?
The bottom of the aquarium, or base of the prism, measures 6 inches by 12 inches, so the area of the base is 72 square inches.



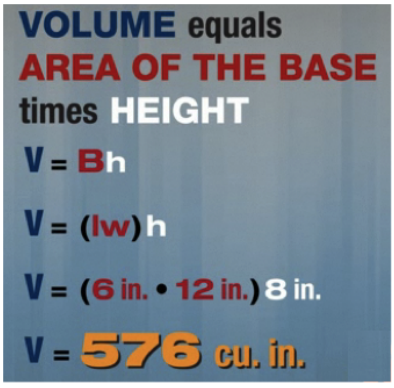
What would the volume of the aquarium be if it were only 1 inch tall?
If the aquarium were only 1 inch tall, the volume would be 72 cubic inches.

It actually takes 8 layers, each with 72 cubes, to fill this aquarium. How many cubes does it take to fill the aquarium? What is the volume of the rectangular prism?



Since each layer is 1 inch tall and the height of the prism is 8 inches, it takes 8 x 72, or 576 cubes, to fill this rectangular prism aquarium. The volume is 576 cubic inches.

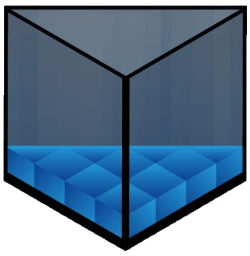
Let's describe what we did with a formula.
We found the volume of this aquarium by taking the area of the base times the height of the prism. We usually represent area of the base with an upper-case B and height with a lower-case h.



For a rectangular prism, the area of the base equals the length of the base times the width of the base. This aquarium measures 6 inches by 12 inches by 8 inches. So the volume is 576 cubic inches.

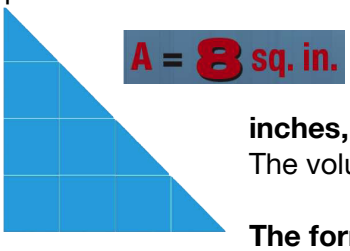
Fish tanks come in all shapes and sizes. How do we find the volume of a triangular prism?
The volume of a triangular prism is found in the same way as the rectangular prism – by multiplying the area of the base by the height of the prism.

As you know, the base of this prism is a triangle. Again, we can cover the base using cubes. How many cubes cover the base of the triangular prism?



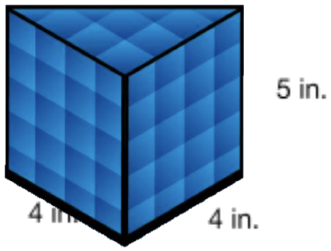
The cubes do not fit exactly like they did in the rectangular prism. It takes 6 full cubes and 4 half cubes to cover the base of this triangular prism. That is a total of 8 cubes.

What is the area of the base of the triangular prism?
Since each cube covers one square inch of the base, the area of the base of the triangular prism is 8 square inches.



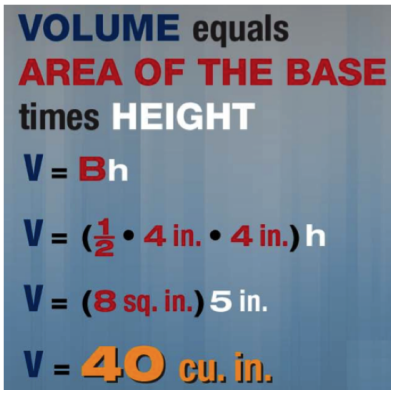
If the aquarium were only 1 inch tall, the volume would be 8 cubic inches, but it takes 5 layers to fill it. What is the volume of the aquarium?
The volume of this triangular prism is 8 x 5, or 40 cubic inches.

The formula, volume equals area of the base times the height, still works. This time the base is a triangle. How do we use the formula to find the volume of this triangular prism?



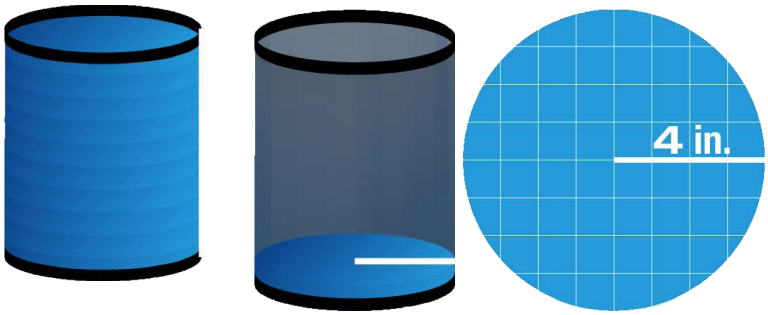
Remember, we find the area of a triangle by taking ½ the base times the height (Area of a triangle = ½bh). So, the area of the base is ½ x 4 (the base of the triangle) x 4 (the height of the triangle).

For this prism, the area of the base is 8 square inches, and the height of the prism is 5 inches. The volume is 40 cubic inches.



Figuring out the volume of a cylinder is very similar to figuring out the volume of a prism. What is the formula for finding the volume of a cylinder?
The formula is still volume equals area of the base times the height. The only difference is that now we need to find the area of a circular base.

What is the volume of this cylinder?



Remember the formula for area of a circle is pi times the radius squared (Area of a circle = πr^2). Let's use 3.14 to approximate pi. The radius of the base is 4 inches. Four squared is 16, so the area of the base is approximately 3.14 x 16 square inches, or about 50 square inches. To approximate the volume, take the area of the base times the height. The height of the cylinder is 10 inches, so the volume is approximately 500 cubic inches.

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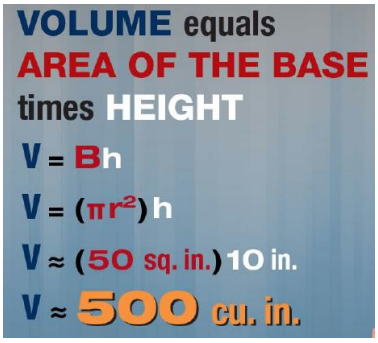
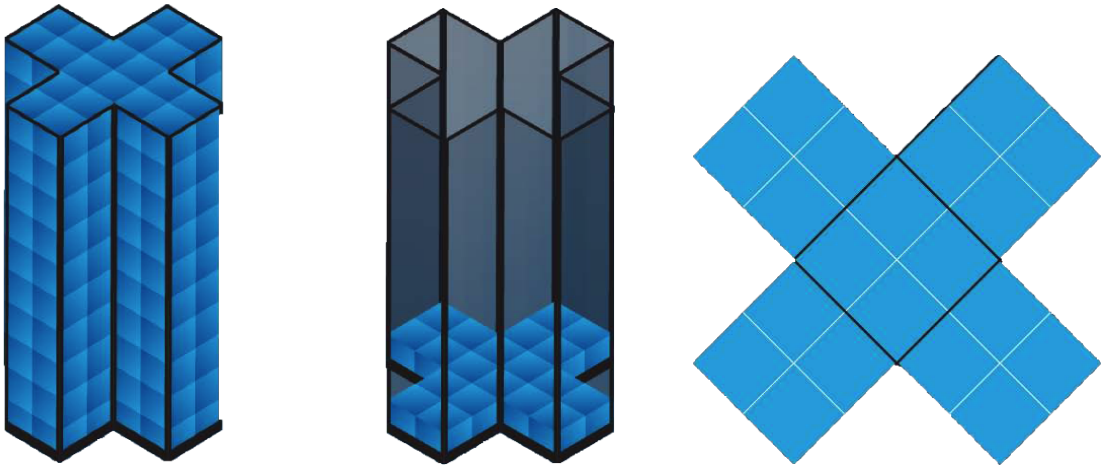


If Kali chooses a tank for her fish that has an odd-shaped base, can she still determine the volume using the same method?

No problem! The formula for finding the volume of any prism or cylinder is always volume equals the area of the base times the height.

The area of this aquarium's base is equal to 20 square inches, and the height is 10 inches. What is the volume of this aquarium?



To find the volume, take 20 square inches times the height which is 10 inches. The volume is 200 cubic inches.

What do you need to know in order to determine the volume of any prism or cylinder?
To determine the volume of any cylinder or prism, you need to know the area of the base and the height. The process of determining the area of the base is different for different-shaped bases. You can determine the area of the base by using the appropriate area formula or by counting. Then, multiply the area of the base times the height of the prism or cylinder to determine the volume.