Solving for Exponents

A Closer Look at the Video

After watching the *Solving for Exponents* video, make sense of the mathematics by reading through the problem situation and solution. Use the comments and questions in bold to help you create tables, graphs, and equations for the situation.

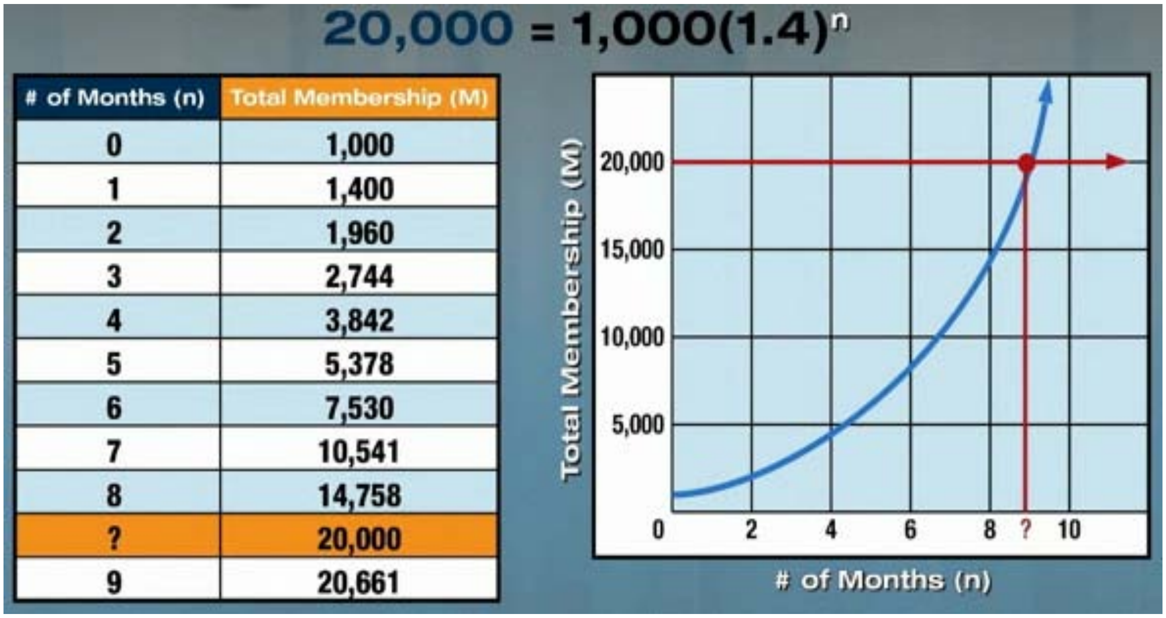
Problem: Live Link Tickets has quickly grown from a good idea into a successful business. The company plans to purchase a new server when their membership reaches 20,000. Live Link Tickets started with a membership of 1,000 and is growing by 40% each month. Determine when the company will need to purchase a new server.

Create a table and graph representing the membership growth.
Live Link Tickets started with a membership of 1,000 and is growing by 40% each month. The team uses the equation $M = 1000(1.4)^n$ to predict total membership for any given month. Here is the table and graph for the equation.

$$M = 1{,}000(1.4)^n$$

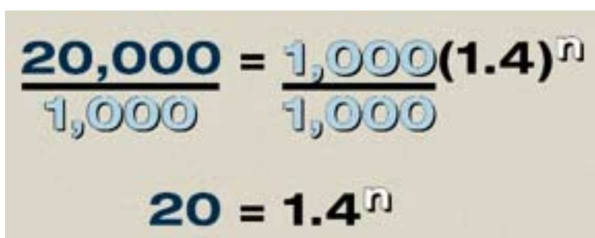
# of Months (n)	Total Membership (M)
0	1,000
1	1,400
2	1,960
3	2,744
4	3,842
5	5,378
6	7,530
7	10,541
8	14,758
9	20,661

How can the team use the table and graph to estimate when they will need to purchase a new server?
They can extend the values for total membership in the table until it exceeds 20,000. They could also get an estimate by looking at the numbers on a graph. The horizontal line shows a membership of 20,000, and the exponential curve shows the total membership when the growth factor is 1.4. The intersection of the curve and line will determine when they need a new server. From the graph and table, we see that the team will need a new server sometime in the eighth month.



What is another method we could use to solve this problem so that the team will know exactly when in the eighth month to get their new server up and running?

We can symbolically solve the equation $20,000 = 1000(1.4)^n$ for n, the unknown number of months. If we divide both sides of the equation by 1000, both sides will remain equal. Then, the equation becomes $20 = 1.4^n$.



How can we solve for the unknown exponent, *n*, in the new equation?

We can determine the value of *n* by using logarithms.

When solving equations we can add or multiply both sides of the equation by the same number or raise both sides to the same power all because of the properties of equality. In this case, we want to take the log base ten to both sides of the equation.

ADD THE SAME NUMBER TO BOTH SIDES OF THE EQUATION

$$x - 10 = 20$$
$$+10 \qquad +10$$
$$x = 30$$

MULTIPLY BOTH SIDES OF THE EQUATION BY THE SAME NUMBER

$$\tfrac{1}{2}x = 10$$
$$2(\tfrac{1}{2}x) = 2(10)$$
$$x = 20$$

RAISE BOTH SIDES OF THE EQUATION TO THE SAME POWER

$$x^{\frac{1}{2}} = 3$$
$$(x^{\frac{1}{2}})^2 = (3)^2$$
$$x = 9$$

Doing this, we get log 20 = log (1.4^n). The right-hand side of the equation is equal to (log 1.4)n. This is true because of a property of logarithms. If you want to learn more about this particular concept, watch *Making Sense of Log Properties*.

We now have log 20 =(log 1.4)n. To finish solving for the number of months, we should divide both sides by log 1.4. This is similar to when we solve the equation 2x = 20; we would divide each side by 2. With our problem, we would divide both sides by the log 1.4. We can use our calculator to divide.

$$20 = 1.4^n$$
$$\log 20 = \log(1.4^n)$$
$$\log 20 = (\log 1.4)\cdot n$$
$$\frac{\log 20}{\log 1.4} = \frac{(\log 1.4)}{\log 1.4}\cdot n$$
$$\frac{\log 20}{\log 1.4} = n$$
$$8.9 \approx n$$

So, it looks like in 8.9 months, Live Link Tickets will have 20,000 members.

How does 8.9 months fit with the estimates we made using the table and graph?
Looking back to our table and graph, we see that the intersection point is a little less than nine months. So, Live Link Tickets needs to purchase a new server sometime during the eighth month as estimated.

MakingSense

$$20{,}000 \approx 1{,}000(1.4)^{8.9}$$

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