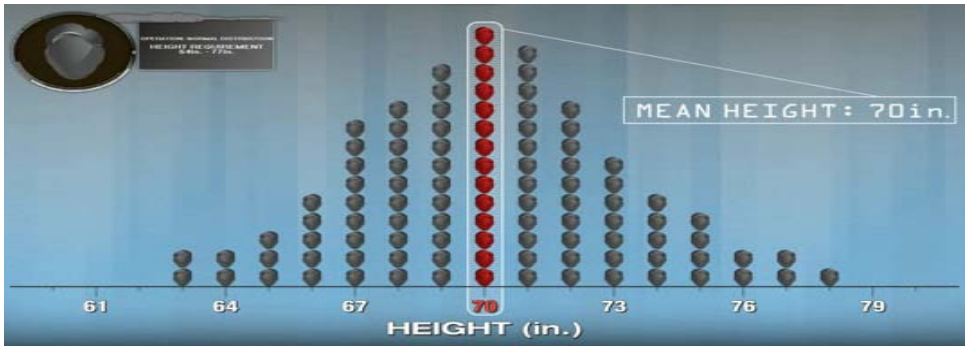
Normal Distribution

A Closer Look at the Video

After watching the *Normal Distribution* video, make sense of the mathematics by reading through the problem situation and solution. Use the comments and questions in bold to help you understand normal distributions.

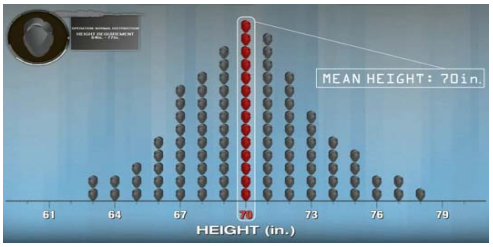
Problem: Tori is touring a flight simulation facility where director Tom Schnau and his team develop support systems for training military pilots. Tom has been explaining all the requirements individuals have to meet to become a pilot. One requirement many people do not think to consider is a person's height. Pilots are restricted to a certain height range because of the design of the cockpits. In order to become a pilot your height must be between 64 and 77 inches. In the United States, the average height of a man is approximately 70 inches, or 5 foot 10, and women average 64 inches, or 5 foot 4. A lot of people want to fly, but Tori wonders what percentage of adults would actually meet the height requirement. Use the normal distribution of the height data to find out.

What information can we gather from a dot plot representing the heights of a random sample of 100 men?

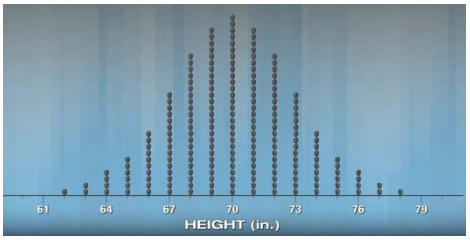


Recall that the average height of a man is 70 inches. Notice many men are shorter than 70 inches while others are taller. The height data is close to being symmetrical about the mean.

What happens to the shape of the dot plot as we gather height data for more men?
As we gather height data for more and more men, the plot is even more symmetrical about the mean.



Data From 100 Men

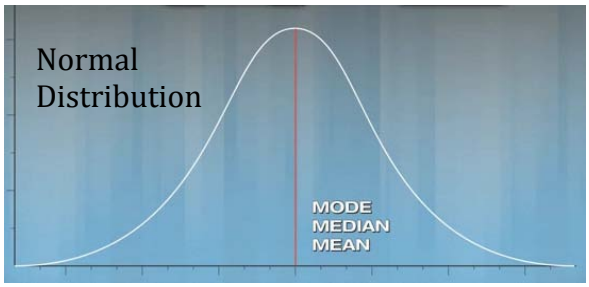


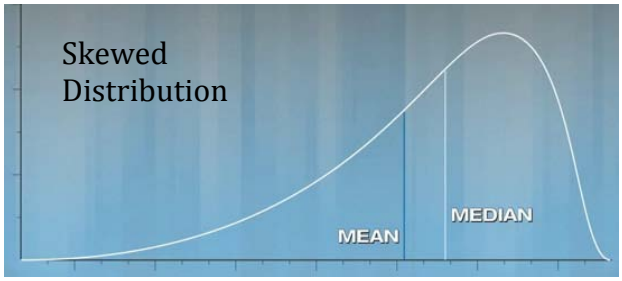
Data From 200 Men

Create a histogram for this data. What happens to the shape of the histogram as we use narrower intervals?
If we use narrower and narrower intervals, we see a bell shape emerge.

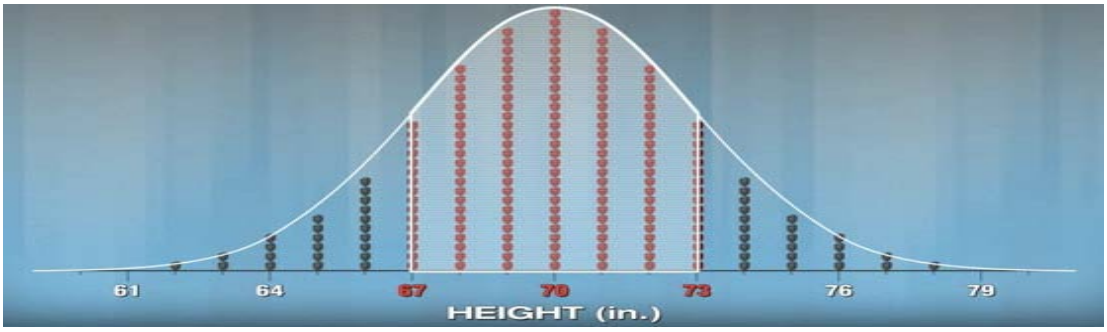
What is a distribution with a bell shape called?
A distribution with this type of bell shape is called a normal distribution.

Due to the symmetric nature of the normal distribution, the values of the mode, median, and mean are the same. Is this true for all distributions?
This is not true for all distributions. When the data is skewed left or skewed right, the mean, median, and mode are not the same, so it is not a normal distribution.



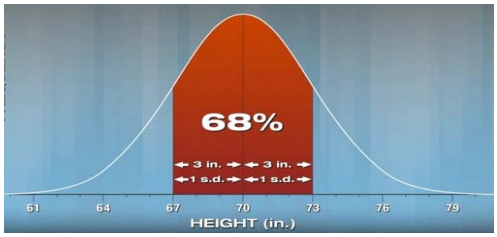


Observe that many men are between 67 and 73 inches tall. About how much of the population is within three inches of the mean?
Remember the average height is 70 inches. It looks like about a third of the population falls between 70 and 73 inches, and another third of the population falls between 67 and 70 inches. So, about two thirds of the population is within three inches of the mean.

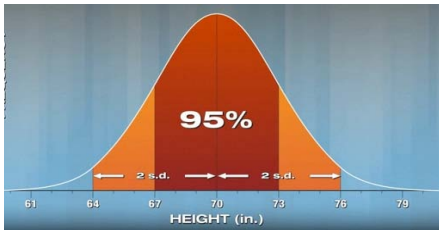


Three inches is an important value for our data set. It is called the standard deviation. What is standard deviation and what is special about the standard deviation of a normal distribution?
The standard deviation of a distribution is a measure of the spread of the data. Every normal distribution has a unique standard deviation and certain percentages of the data fall within one, two, and three standard deviations from the mean.

It turns out that for every normal distribution, 68% of the data fall within one standard deviation of the mean. So, our estimate was right; a little more than two thirds of the men are between 67 and 73 inches tall.



What happens within two standard deviations of the mean?
That takes us down to 64 inches and up to 76 inches. About 95% of the population lies within two standard deviations of the mean. Remember, when we say within two standard deviations of the mean, we are referring to two standard deviations above the mean, and two standard deviations below mean.

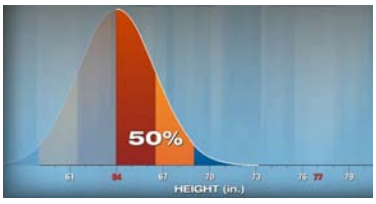


Finally, what happens within three standard deviations above the mean, up to 79 inches, and three standard deviations below the mean, down to 61 inches?
99.7% percent of men's heights are within this three standard deviation interval.

Recalling that to be a pilot your height must be between 64 and 77 inches, about what percentage of men meet the height requirement for becoming a pilot?
The range 64 to 77 inches is close to our two standard deviation interval, which means slightly more than 95% of men meet the height requirement for pilots.

Women's heights are also normally distributed, but the mean is 64 inches. If the standard deviation is the same as the men's, the shape of the distribution would be the same but centered about the mean of 64 inches instead of 70 inches. **If the standard deviation is larger than the men's, then the shape of the distribution would widen. If the standard deviation is smaller than the men's, then the shape of the distribution would narrow.** According to current research, the standard deviation for the height of women is approximately two and a half inches, so the shape of the normal distribution is slightly narrower.

What percentages of women meet the height requirement to be a pilot?
Any woman who is between 64 inches (5 foot 4 inches) and 77 inches (6 foot 5 inches) meets the height requirement. Notice that the minimum height is the average height of a woman and very few women are taller than 77 inches, the maximum height. So, we can say about 50 percent of women meet the height requirement.



Today, we used the normal distribution to look at heights, but many sets of data, like standardized test scores, the weight of a newborn baby, and the life of a cell phone battery, are normally distributed. What features define all normal distributions?

- Bell shape curve
- Symmetric about the mean
- 68% of the data fall within one standard deviation of the mean
- 95% of the data fall within two standard deviations of the mean
- 99.7% of the data fall within three standard deviations of the mean