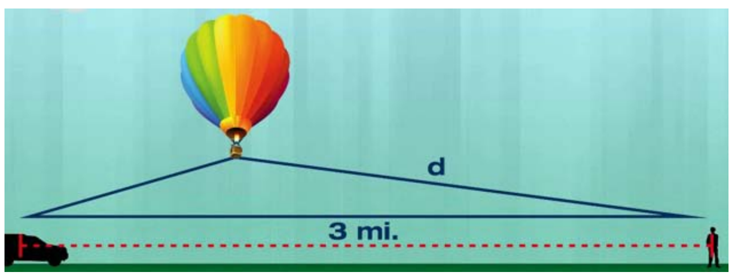
Law of Sines

A Closer Look at the Video

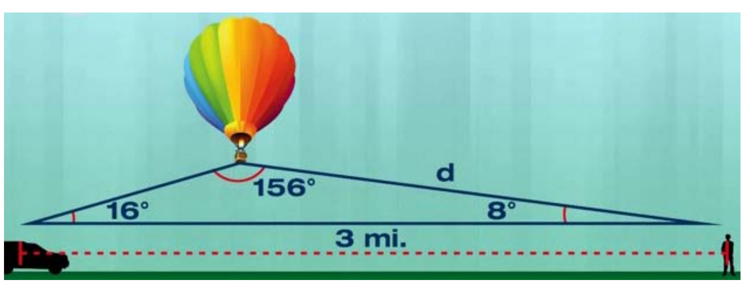
After watching the *Law of Sines* video, make sense of the mathematics by reading through the problem situation and solution. Use the comments and questions in bold to help you understand the Law of Sines.

Problem: Skylar's friend, Doug, is taking a hot air balloon ride, and Skylar is helping out at the launch site. While up in the balloon, Doug takes a picture of Kyle, another friend, who is waiting at the landing point to help safely lower the balloon. Doug wants to know how far away from Kyle he is when he takes the picture. Use the Law of Sines and help Skylar and his friends determine how far Doug is from Kyle.

The launch point, the balloon, and Kyle form a triangle, as illustrated below. Observe that the distance between Kyle, at the landing site, and Skylar, at the launch site, is 3 miles.



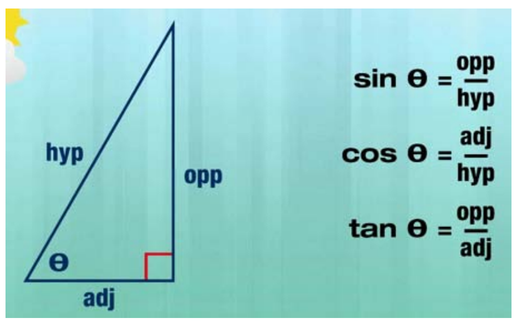
Using clinometer applications on their phones, Skylar and Kyle determine that the angles of elevation from the launch point and landing point are 16° and 8° respectively. What is the measure of the third angle in the triangle?
By subtracting the measures of the two angles from 180°, we can find that the third angle is 156°.



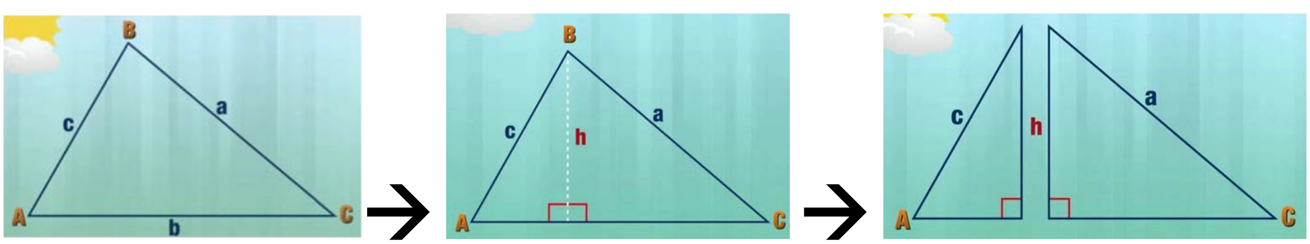
So, we know all three angles but we only know one side length. What do we need to know to determine if this is enough information to figure out the distance between the balloon and Kyle?
We need to know the relationship between the lengths of the sides of our triangle and the measures of its angles.

What is a good approach to use when trying to find missing side lengths or angle measures in a triangle?
Frequently, when we are trying to find a missing side length or angle measure of a triangle, a good approach is to use right triangles. Right triangles tend to be easier to deal with because we know a lot about them.

From studying right-triangle trigonometry you may recall sine, cosine, and tangent. What are the sine, cosine, and tangent of an angle?
Sine, cosine, and tangent are ratios of the side lengths of a right triangle. The sine of an angle is found by dividing the length of the side opposite the angle by the length of the hypotenuse. Cosine is the length of the adjacent side divided by the length of the hypotenuse. Tangent is the length of the opposite side divided by the length of the adjacent side. Notice all of these functions are ratios of side lengths.



Given any triangle, we can divide it into two right triangles, as shown. How can we use the sine ratio to relate the side lengths *a*, *c*, and *h* and the angles *A* and *C*?

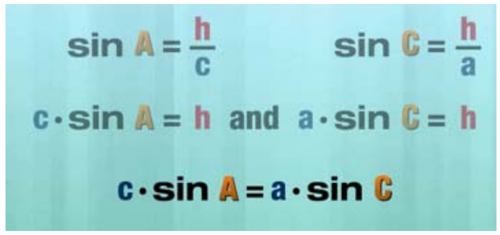


The sine of *A* is equal to *h* divided by *c*, and the sine of *C* is *h* divided by *a*.

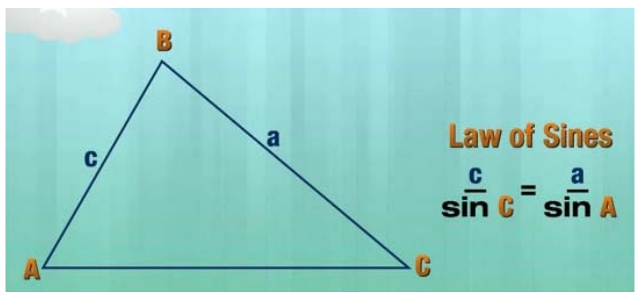
$$\sin A = \frac{h}{c} \qquad \sin C = \frac{h}{a}$$

How can we combine these two equations into one equation?

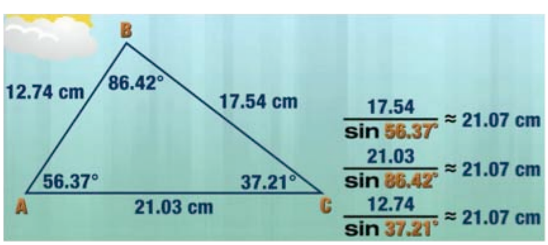
Notice the height, h, is a variable in both of these equations. If we solve for the height, we get $c \cdot \sin A = h$ and $a \cdot \sin C = h$. Since $c \cdot \sin A = h$ and $a \cdot \sin C = h$, we have $c \cdot \sin A = a \cdot \sin C$.

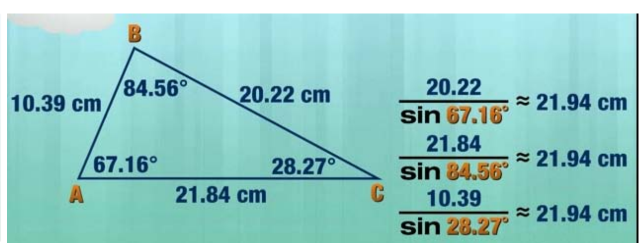


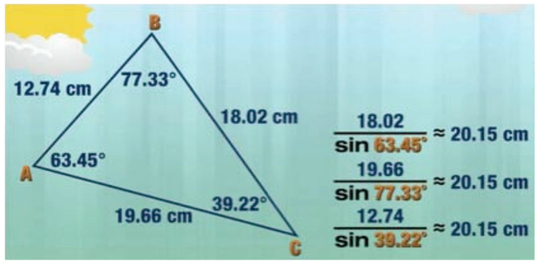
Rearranging the equation, we get the length of side c divided by sin C is equal to the length of side a divided by sin A. This is called the Law of Sines. Here is a cool thing, after all that algebra, h is not even part of the equation any more.



What does the Law of Sines mean?
The Law of Sines means that for any triangle, if you divide any side length by the sine of its opposite angle and divide another side length by the sine of its opposite angle, you get the same value. Notice that while the side lengths and angles change, the ratios remain constant. So, there is a relationship between the sines of the angle measures and the lengths of the opposite sides.







Let's see how the relationship we just found can help us. How can we use the Law of Sines to find the distance between the balloon and Kyle?
Remember the landing point is 3 miles away from the starting point. As we found earlier, its opposite angle measures 156°. Setting up a proportion using the Law of Sines, we see that 3 divided by the sine of 156 is equal to the distance to the landing, *d*, divided by the sine of 16. We use 16° because it's the angle opposite to the distance we are trying to find. Multiplying both sides by the sine of 16, we get the sine of 16 times 3 divided by the sine of 156. Calculating this equation tells us that the picture of Kyle was taken from 2 miles away.

$$\frac{3}{\sin 156} = \frac{d}{\sin 16}$$

$$\sin 16 \cdot \frac{3}{\sin 156} = \frac{d}{\sin 16} \cdot \sin 16$$

$$\frac{\sin 16 \cdot 3}{\sin 156} = d$$

$$2 \text{ mi.} \approx d$$

The Law of Sines is used to find angle measures and side lengths in triangles. So, when can we use it?
You can use the Law of Sines when you know an angle measure and its opposite side length plus one additional angle measure *or* one additional side length.

Let's review. What is the Law of Sines?
The Law of Sines is a proportional relationship involving side lengths and the sine of the opposite angle; *a* divided by sin *A* is equal to *b* divided by sin *B* and *c* divided by sin *C*.