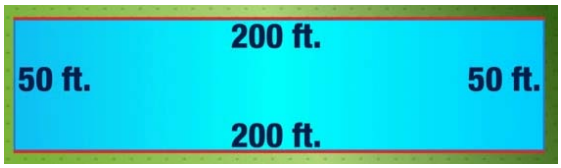
PROBLEM SOLVED
MAKING SENSE OF MATHEMATICS

# Fixed Perimeter, Changing Area
## A Closer Look at the Video

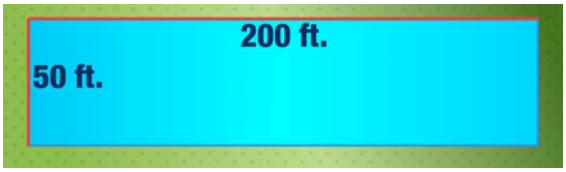
After watching the *Fixed Perimeter, Changing Area* video, make sense of the mathematics by reading through the problem situation and solution. Use the comments and questions in bold to help you understand how to find the maximum area for a fixed perimeter.

**Problem:** *Al is installing an underground dog fence for his dog, Stella. He wants to provide Stella with plenty of room to run but still keep her within his yard. The system Al purchased includes 500 feet of fence, so the perimeter, or total length of the fence, is 500 feet. Help Al determine the dimensions for a rectangular region that will allow Stella to have as much area as possible.*

**Al started by exploring some possible rectangles with a fixed perimeter of 500 feet. First he made the width 50 feet. What is the length of this rectangle?**
Since the width is 50, the two sides corresponding to the width will use 100 feet of fencing. With 400 feet of fencing left, each of the sides corresponding to the length must be 200 hundred feet.

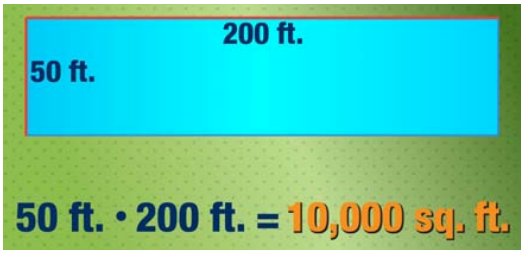



**What is another way to determine the length of the rectangle with a fixed perimeter of 500 feet and a width of 50 feet?**
The sum of the width and the length of the rectangle is equal to half the perimeter. So, if the perimeter is 500 feet, half of this is 250 feet. This means the length plus the width is 250 feet, or the length is 250 – 50 = 200 feet.




Together these two sides measure 250 feet, which is half the perimeter.

**What is the area of this rectangle?**
As you know, the area of a rectangle is found by multiplying width and length. Fifty feet times 200 feet equals 10,000 square feet.

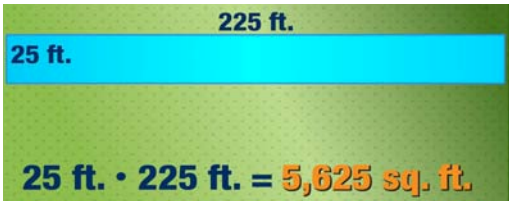



**Next, Al tried making the width 25 feet. What is the length of this rectangle?**
We know that half of the perimeter is 250 feet, so the length must be 250 – 25 = 225 feet.

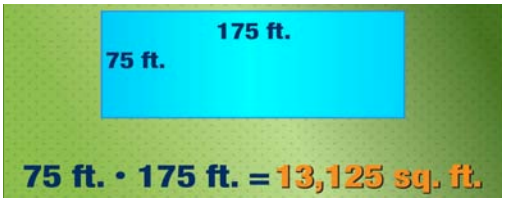



MakingSense

**What is the area of this rectangle?**
The area is 5,625 square feet. Notice, the perimeter is still 500 feet, but the area is smaller than the area of the first rectangle.




**What is the area if the width is 75 feet and the length is 175 feet? How does this compare to the previous areas found.**
The area is 13,125 square feet. This is considerably larger than the two previous areas.




**Is this the largest possible area? If we just keep guessing, Stella could be full-grown before we figure out the fencing. What is a more efficient way to make sense of how the area changes as the width changes?**
We can organize our work by putting the information into a table like the one below.

| Perimeter | Width | Length | Area |
|---|---|---|---|
| 500 ft. | 25 ft. | 225 ft. | 5,625 sq. ft. |
| 500 ft. | 50 ft. | 200 ft. | 10,000 sq. ft. |
| 500 ft. | 75 ft. | 175 ft. | 13,125 sq. ft. |

**The next rectangle Al tried measured 100 feet by 150 feet, so its area is 15,000 square feet. Do you notice the pattern?**

| Perimeter | Width | Length | Area |
|---|---|---|---|
| 500 ft. | 25 ft. | 225 ft. | 5,625 sq. ft. |
| 500 ft. | 50 ft. | 200 ft. | 10,000 sq. ft. |
| 500 ft. | 75 ft. | 175 ft. | 13,125 sq. ft. |
| 500 ft. | 100 ft. | 150 ft. | 15,000 sq. ft. |

The closer the measurements of the width and the length are to each other, the larger the area.

**What would happen if the width and the length were equal? That would be a square. Is a square a rectangle?**
A rectangle is a quadrilateral with four right (90°) angles. A square has four right angles, so a square is also a rectangle.

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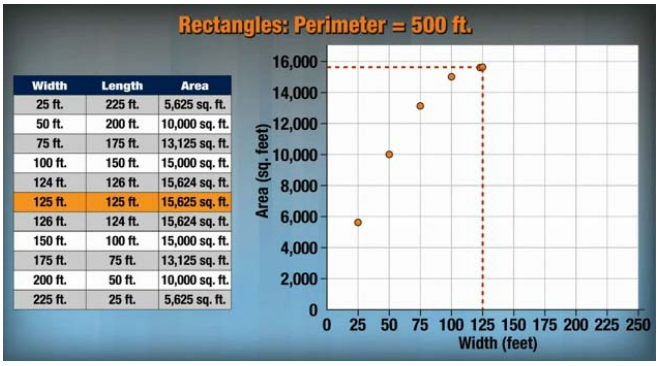
**What is the area of the square? How does it compare to the other areas we have found?**
The area of the square is 125 x 125 = 15,625 square feet. That is the largest area we have found.

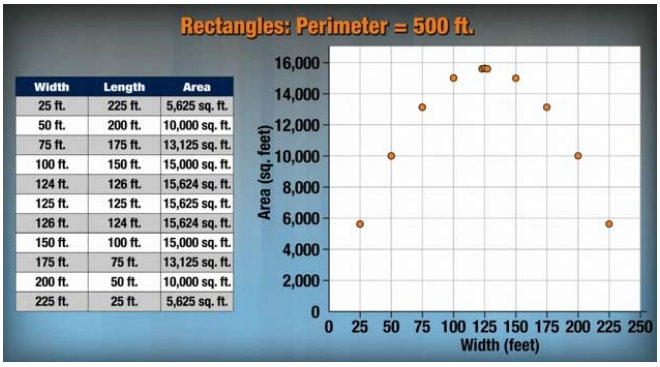
**Could there be a rectangle with a larger area?**
A width of 126 feet gives an area that is smaller than the area of the square. A rectangle measuring 124 feet by 126 feet has the same area. This rectangle is congruent to the one measuring 126 feet by 124 feet. Switching the dimensions of a rectangle does not change the area.

**Rectangles: Perimeter = 500 ft.**

| Width | Length | Area |
|---|---|---|
| 25 ft. | 225 ft. | 5,625 sq. ft. |
| 50 ft. | 200 ft. | 10,000 sq. ft. |
| 75 ft. | 175 ft. | 13,125 sq. ft. |
| 100 ft. | 150 ft. | 15,000 sq. ft. |
| 124 ft. | 126 ft. | 15,624 sq. ft. |
| 125 ft. | 125 ft. | 15,625 sq. ft. |
| 126 ft. | 124 ft. | 15,624 sq. ft. |
| 150 ft. | 100 ft. | 15,000 sq. ft. |
| 175 ft. | 75 ft. | 13,125 sq. ft. |
| 200 ft. | 50 ft. | 10,000 sq. ft. |
| 225 ft. | 25 ft. | 5,625 sq. ft. |

**Is there another way to make sense of how the area changes as the width changes?**
We can also make sense of this by looking at a graph. The x-axis is the width of the rectangle and the y-axis is the area. Graphing the results allows us to see how the total area increases as the width increases to the point of maximum area. After that point, the area decreases even though the width increases.




**Our graph just shows some rectangles with a perimeter of 500 feet. What would a graph of all possible rectangles look like?**
If we show all possible rectangles, our graph would be a continuous curve like the graph shown below.

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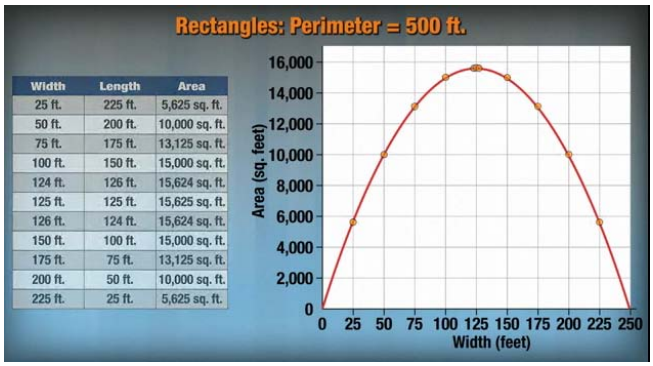


| Width | Length | Area |
|---|---|---|
| 25 ft. | 225 ft. | 5,625 sq. ft. |
| 50 ft. | 200 ft. | 10,000 sq. ft. |
| 75 ft. | 175 ft. | 13,125 sq. ft. |
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| 124 ft. | 126 ft. | 15,624 sq. ft. |
| 125 ft. | 125 ft. | 15,625 sq. ft. |
| 126 ft. | 124 ft. | 15,624 sq. ft. |
| 150 ft. | 100 ft. | 15,000 sq. ft. |
| 175 ft. | 75 ft. | 13,125 sq. ft. |
| 200 ft. | 50 ft. | 10,000 sq. ft. |
| 225 ft. | 25 ft. | 5,625 sq. ft. |

**How many possible rectangles are there?**
When trying out different rectangles before, we only considered widths that were whole number values, which gave us whole number values for the length as well. But, we could have chosen any real number value from 0 to 250 feet for the width. For example, we could have a rectangle that measures 10½ feet by 239½ feet. There are actually an infinite number of rectangles possible. All of these rectangles have the same perimeter but different areas.

**What happens to the area of the fenced-in yard as the dimensions get closer to that of a square? What happens to the area when the rectangle is less square?**
The area of our fenced in yard gets larger as we make it more like a square. The longer and thinner the rectangle is, the smaller the area.