PROBLEM SOLVED
MAKING SENSE OF MATHEMATICS

Area of a Triangle
A Closer Look at the Video

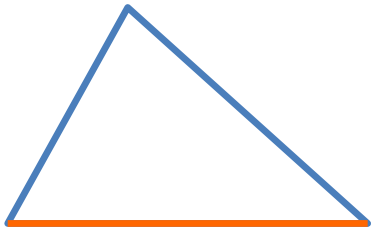
After watching the *Area of a Triangle* video, make sense of the mathematics by taking a closer look at the problem situation and solution. Use the questions and comments in bold to help you understand the formula for the area of a triangle.

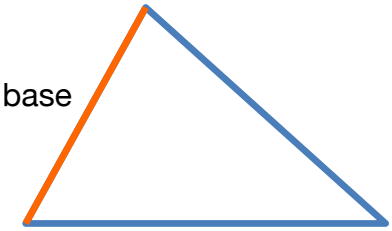
Problem: The band, the Trilaterals, is getting ready to go on tour. They are designing the stage with a riser for the drum kit. The riser is big and in the shape of a triangle, and the drummer wants to cover the top of the triangle with metal. The band needs to determine the area of the triangle, so they can purchase the correct amount of metal covering. If a base of the triangle is 18 feet and its corresponding height is 6 feet, how many square feet of metal does the band need to purchase to cover the riser?

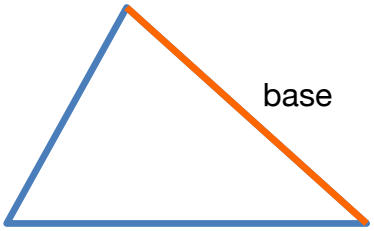
What two key elements must you know about a triangle in order to determine its area?
To determine the area of a triangle, you must know

- the length of a base of the triangle and
- the height of the triangle corresponding to the chosen base.

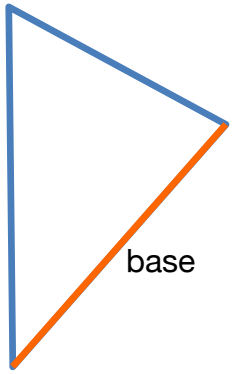
How do you know which side of the triangle is the base? A triangle has three sides, and any of the three sides can be the base.







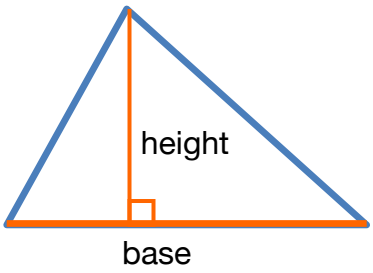
The triangle does not have to sit on its base.

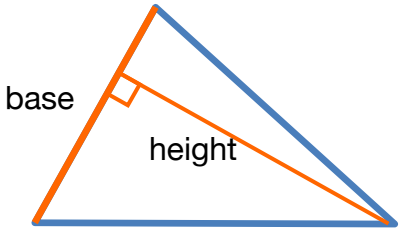


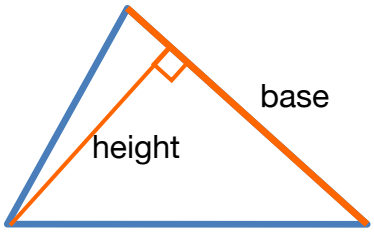
How do you determine the height of a triangle? The height of a triangle tells you the distance from the base to the vertex opposite that base. No matter which base you choose,

MakingSense

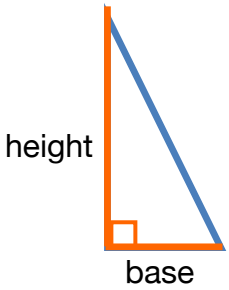
the height is always perpendicular to the base. That means the base and height form a right angle.



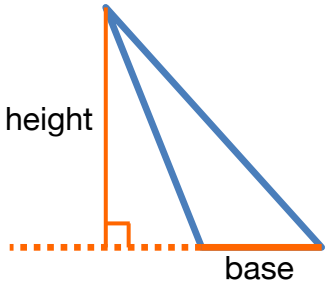




The height of a triangle can also be the side of the triangle if the side is perpendicular to the base. When this is the case, the triangle is a right triangle.

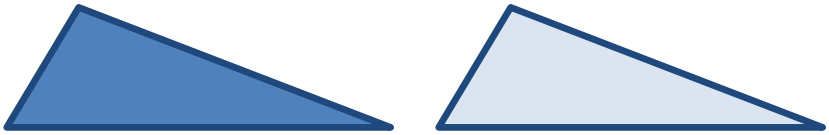


When the triangle is obtuse, it has one angle greater than 90 degrees, and the height for the base may fall outside the triangle as shown below.



In order to make sense of the formula for the area of a triangle, consider the formula for area of a parallelogram. The area of a parallelogram is determined by taking the base times the height ($A = b \cdot h$).

The following two triangles are congruent; they are exactly the same shape and size. How might you place the two triangles together to form a parallelogram?



Area of a Triangle

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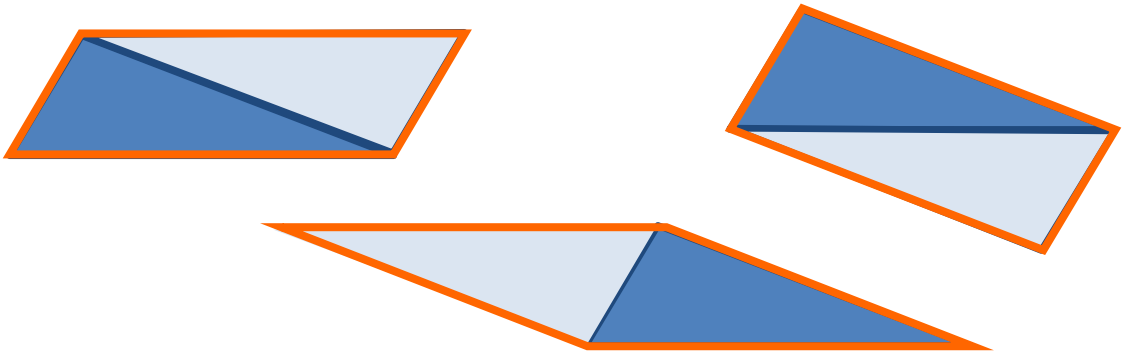
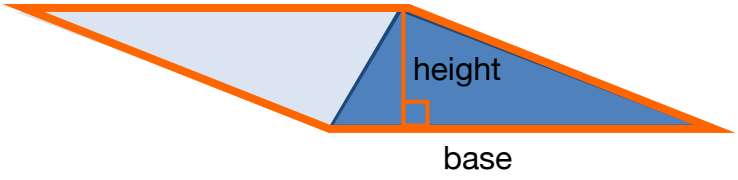
You can place two congruent triangles together to form a parallelogram in several ways.



Compare the parallelogram formed by placing the two triangles together to the original triangle. How does the base of the triangle compare to the base of the parallelogram? They are the same. **How does the height of the triangle compare to the height of the parallelogram?** They are also the same. **How does the area of the triangle compare to the area of the parallelogram?** Since the parallelogram has the same area as two congruent triangles, the area of one triangle is one half the area of the parallelogram.



How can you use this information to determine the formula for area of a triangle?
The formula for area of a parallelogram is base times height. The area of one triangle is one half the area of the parallelogram so the formula for the area of a triangle is ½(b • h). This is true for any triangle because you can always place two congruent triangles together to form a parallelogram with the same base and height.

Area of a parallelogram = b • h

Area of a triangle = $\frac{1}{2}$ (b • h)

The triangle shown below is the shape of the drum riser. How much metal does the band need to purchase to cover the riser?

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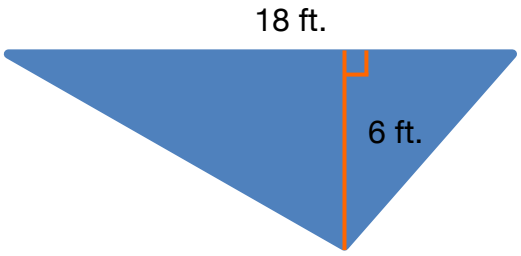
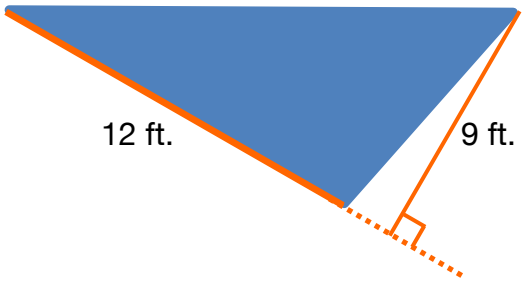


$$\text{Area of a triangle} = \frac{1}{2}(b \cdot h)$$
$$= \frac{1}{2}(18 \cdot 6)$$
$$= 54 \text{ sq. ft.}$$

Does the area of a triangle change if a different side of the triangle is used as the base? Find the area of the triangle again, using the base and height shown below.



$$\text{Area of a triangle} = \frac{1}{2}(b \cdot h)$$
$$= \frac{1}{2}(12 \cdot 9)$$
$$= 54 \text{ sq. ft.}$$

The area of a triangle does not change when you use a different side as the base and its corresponding height to do the calculations. You can find the area of any triangle by selecting one side for the base and using the formula $A = \frac{1}{2}(b \cdot h)$.