PROBLEM SOLVED
MAKING SENSE OF MATHEMATICS

Exponential Decay
A Closer Look at the Video

After watching the *Exponential Decay* video, make sense of the mathematics by reading through the problem situation and solution. Use the comments and questions in bold to help you create tables and equations for the situation.

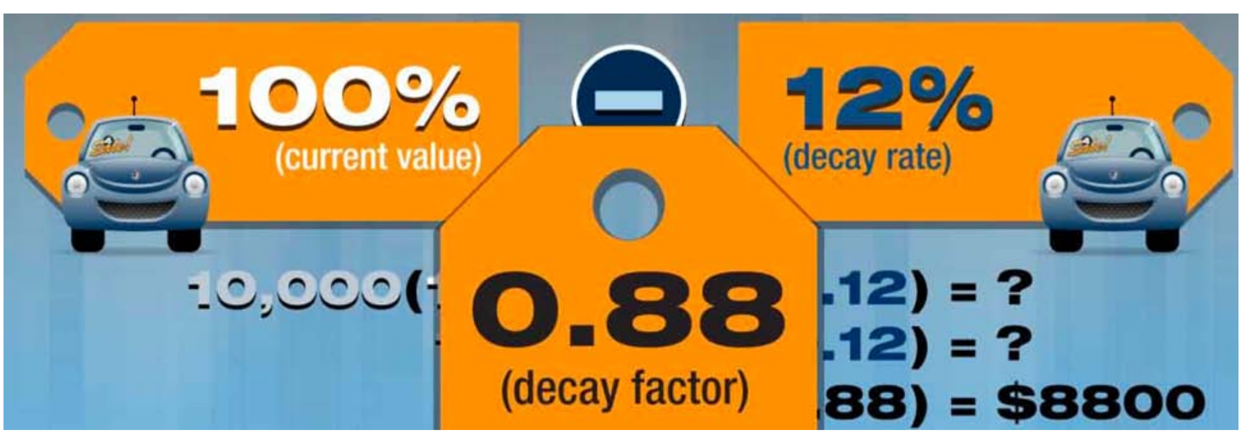
Problem: Jill is in the market for her first car. In a few years she hopes to trade in her first car for the car of her dreams. When making her selection, Jill will have to keep resale value in mind and understand how cars depreciate. If she buys a car worth $10,000 now, and its value decreases by 12% per year, how much will this car be worth in ten years?

The car is depreciating at a rate of 12% per year. What does that mean?
It means that a year from now the car's value will decrease by 12% or 0.12 of the starting value, $10,000. That is in one year, the car will be worth $1,200 less than it is worth now.

How can you write an equation to find the value of the car at the end of one year?
Take 100% of the original value and subtract 12% of the original value. So, 10,000(100%), or 10,000(1), minus 10,000(12%) or 10,000(0.12) is equal to 10,000(1 - 0.12) or 10,000(0.88), which equals $8,800, the value one year from now.

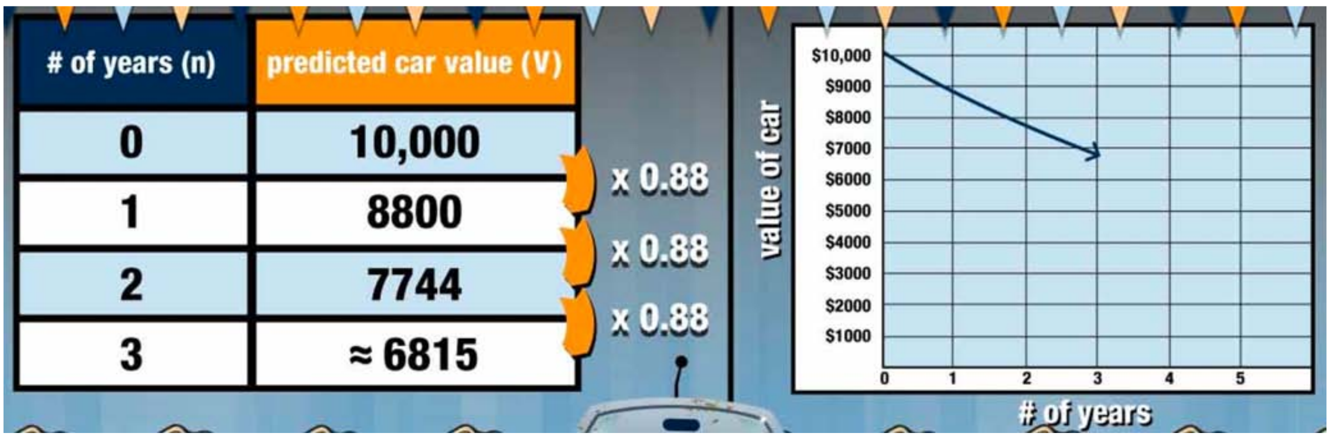
$$10{,}000(1) - 10{,}000(0.12) = ?$$
$$10{,}000(1 - 0.12) = ?$$
$$10{,}000(0.88) = \$8800$$

12% or 0.12 is the decay rate, but the decay factor is 1 – 0.12, which equals 0.88 or 88%. **The decay factor is the amount of value the car maintains from year to year. The decay factor is always between zero and one.**



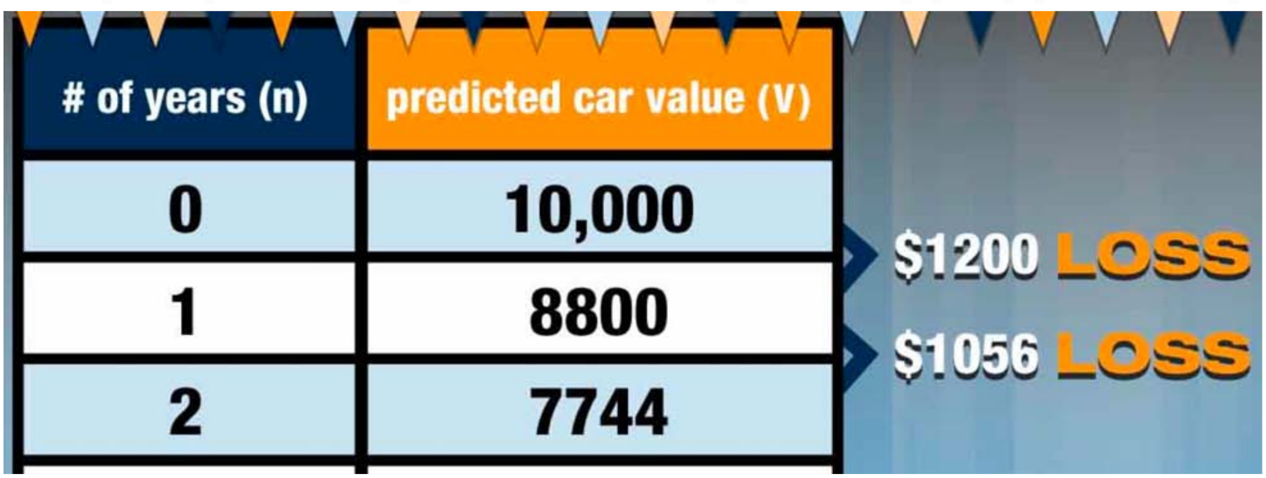
MakingSense

How can we use the decay factor of 0.88 to find the value of the car at the end of Year 2? To find the value at the end of the second year, we take the estimated value of the car after the first year, $8,800, and multiply it by the decay factor of 0.88, which equals $7,744. If we continue the process of multiplying our new value by the decay factor, we predict the value will be approximately $6,815 by the end of the third year.



# of years (n)	predicted car value (V)
0	10,000
1	8800
2	7744
3	≈ 6815

Notice, during the first year, the car value is predicted to depreciate by $1,200, but during the second year, the car's value is predicted to depreciate by <u>less</u> than $1,200. **The depreciation is not the same from year to year.**



# of years (n)	predicted car value (V)
0	10,000
1	8800
2	7744

Jill wants to find the value of the car in ten years. She could continue to multiply by 0.88 for each of the ten years**. What is another way we can predict the car's value after ten years?**

Since we repeatedly multiply by the decay factor, we can use exponents.
We started with $10,000 and multiplied by our decay factor of 0.88. Then, to find the estimated value of the car at the end of the second year, we multiplied our value at the end

of Year 1 by 0.88. Remember, 10,000(0.88) = 8,800. So, to help us find a pattern, we replace 8,800 with 10,000(0.88). For the third year, we multiply the second year's value by 0.88. Let's write 7,744 as 10,000(0.88)(0.88).

# of years (n)	predicted car value (V)	
0	10,000	10,000
1	10,000(0.88)	8800
2	10,000(0.88)(0.88)	7744
3	10,000(0.88)(0.88)(0.88)	≈ 6815

Now we can begin to see the pattern. **The exponent corresponds to the number of years, which is represented by n.** Exponents can help us calculate the predicted value for any year; we simply take 10,000 times 0.88 raised to the nth power. If we want to predict what the car's value would be in ten years, we take $\$10,000(0.88)^{10}$, which is approximately $2,785.

# of years (n)	predicted car value (V)	
0	10,000 = $10,000(0.88)^0$	10,000
1	10,000(0.88) = $10,000(0.88)^1$	8800
2	10,000(0.88)(0.88) = $10,000(0.88)^2$	7744
3	10,000(0.88)(0.88)(0.88) = $10,000(0.88)^3$	≈ 6815
n	$10,000\underbrace{(0.88)...(0.88)}_{n} = 10,000(0.88)^n$	

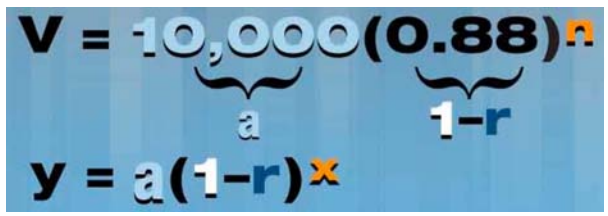
How can we calculate decay for any situation? We need two important values:

- the value of the dependent variable when *n* equals 0, which can be represented by *a*.
- the decay factor, which can be represented by *1 - r*.

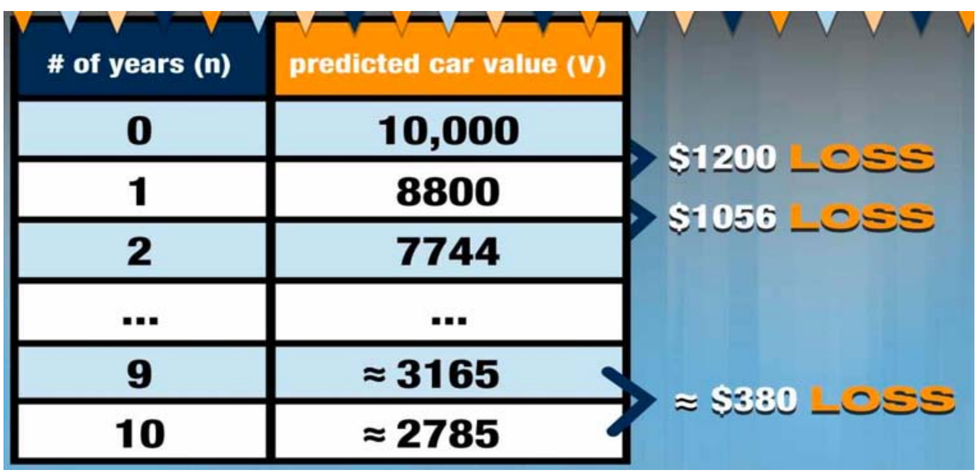
What are the two important values for our problem?

- *a* is the value of the dependent variable when $n = 0$.
  - In our equation this value is 10,000 since $10{,}000(0.88)^0 = 10{,}000$.
- the decay factor.
  - In our equation, the decay factor is 0.88 or one minus the decay rate of 0.12.

How can we generalize our exponential decay equation?
Instead of using *V* for value and *n* for number of years, we can replace these variables with *y* and *x*. So, any exponential decay equation can be represented by $y = a(1- r)^x$.



Here's something to remember. **The greatest decay always occurs right away and then lessens over time.** In the first year, the car's value will decrease by a whopping $1,200, but in the tenth year, we're predicting it will only decrease by about $380.



# of years (n)	predicted car value (V)
0	10,000
1	8800
2	7744
...	...
9	≈ 3165
10	≈ 2785