PROBLEM SOLVED
MAKING SENSE OF MATHEMATICS

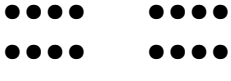
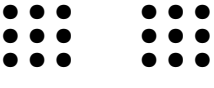
Difficult Addition Facts
Two-week Learning Unit

Directions for Parents and Tutors: The following mini-lessons focus on helping children learn the difficult addition facts by using two different thinking strategies. The first strategy is called *Use Doubles*, and the second strategy is called *Use Ten*. Both strategies help children determine the answer to an unknown basic fact by using facts they already know – doubles and facts that add to ten.

Use the following 10 mini lessons over the course of two weeks. Spending 10 to 15 minutes each day over two weeks is much more effective than a few hour-long sessions.

Day 1 – Addition Doubles

Verbally ask your child to determine the sum of each of the following doubles. If your child does not respond with a correct answer within a few seconds, or if he or she counts to get the answer, represent the double with items or a drawing. The table below lists sample items you might use to represent each double. If these items are not available, draw a sketch of the items or a sketch of dots. You may also represent the doubles with dominoes or a pair of dice. Talk about the double and determine the answer together. Note which doubles were difficult for your child and review those problems periodically over the course of the day. Look for real-life examples of doubles as you go about your daily activities and use the examples to help your child learn the addition doubles.

Double	Representation with items	Dot Representation
5 + 5 = 10	• Fingers on two hands • Toes on two feet	
4 + 4 = 8	• legs on a spider • hamburger buns in a package • two 4-leaf clovers	
6 + 6 = 12	• eggs in a carton • cans of soda in a 12-pack	
1 + 1 = 2	• pair of shoes • pair of gloves	
8 + 8 = 16	• legs on two spiders • tentacles on two octopuses	
2 + 2 = 4	• rolls of toilet paper in a package • pudding cups in a package • wheels on a car	
9 + 9 = 18	• wheels on a semi-truck (18 wheeler) • eggs in a large carton (1½ dozen)	
3 + 3 = 6	• six-pack of soda • two cans of tennis balls	
7+ 7 =14	• days in two weeks (look at a calendar)	

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Day 2 – Doubles plus or minus 1

The following table includes basic addition facts that are doubles and facts whose sum is one more or one less than a double. Verbally ask your child to determine the sum of the doubles in column one. Then ask your child to determine the sum of the fact in column two and how he or she determined the answer. The third column describes two efficient reasoning strategies children might use to determine the sum of the fact by using doubles. It is not necessary for a child to describe both ways to think about each fact.

Double	Double + or - 1	Thinking
5 + 5 = 10 6 + 6 = 12	5 + 6 = 11	• 5 + 5 is 10, so 5 + 6 is one more or 11 • 6 + 6 is 12, so 5 + 6 is one less or 11
4 + 4 = 8 5 + 5 = 10	5 + 4 = 9	• 4 + 4 is 8, so 5 + 4 is one more or 9 • 5 + 5 is 10, so 4 + 5 is one less or 9
9 + 9 = 18 8 + 8 =16	9 + 8 = 17	• 9 + 9 is 18, so 8 + 9 is one less or 17 • 8 + 8 is 16, so 8 + 9 is one more or 17
2 + 2 = 4 3 + 3 =6	2 + 3 = 5	• 2 + 2 is 4, so 2 + 3 is one more or 5 • 3 + 3 is 6, so 2 + 3 is one less or 5
7 + 7 = 14 8+ 8 =16	8 + 7 = 15	• 7 + 7 is 14, so 8 + 7 is one more or 15 • 8 + 8 is 16, so 8 + 7 is one less or 15
3 + 3 = 6 4 + 4 =8	3 + 4 = 7	• 3 + 3 is 6, so 3 + 4 is one more or 7 • 4 + 4 is 8, so 3 + 4 is one less or 7
8 + 8 = 16 9 + 9 =18	8 + 9 = 17	• 8 + 8 is 16, so 8 + 9 is one more or 17 • 9 + 9 is 18, so 8 + 9 is one less or 17
6 + 6 = 12 7+ 7 =14	7 + 6 = 13	• 6 + 6 is 12, so 7 + 6 is one more or 13 • 7 + 7 is 14, so 7 + 6 is one less or 13

Look for real-life examples of doubles and doubles plus or minus one as you go about your daily activities. Use the examples to create problems to discuss with your child.

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Day 3 – Doubles plus or minus 1

Each of the following word problems involves an addition or subtraction fact. Verbally ask your child to determine the answer to the problem and how he or she determined the answer. If your child is unsure of how to solve the problem, ask questions to help him or her think about the structure of the problem. The structure of addition and subtraction problems is different. If you know two or more parts, you can add to find the whole (how many in all). If you know one part and the whole (how many in all), you can subtract to find the other part. Sample questions using this language are included after the first two word problems.

1. Sue looks in the refrigerator and finds 4 apples and 3 oranges. How many pieces of fruit is that?
   Sample Questions:
   - How many oranges did Sue find? (3)
   - Is that part of the fruit or all of the fruit? (part)
   - How many apples did Sue find? (4)
   - Is that all of the fruit or part of the fruit? (part)
   - How can you determine how many pieces of fruit there are altogether? (add the parts)
   - What fact do you know that could help you find the answer to 3 + 4? (3 + 3 or 4 + 4)
   - How would you use that fact? (I know 3 + 3 = 6 so 4 + 3 is one more or 7.)

2. Lacrisha and her friend purchased 16 apples. They used 8 apples to make a pie. How many apples are left?
   Sample Questions:
   - How many apples did they buy? (16)
   - Is that part of the apples or all of the apples? (all)
   - How many apples did they use for the pie? (8)
   - Is that all of the fruit or part of the fruit? (part)
   - How can you determine how many apples are left? (subtract part of the apples from all of the apples, 16 – 8 = ___ , or think part of the apples plus part of the apples equals all of the apples, 8 + __ = 16)
   - What fact do you know that could help you find the answer? (8 + 8 = 16)
   - How would you use that fact? (If 8 + 8 = 16, then 16 minus 8 is 8.)

3. There are 8 girls and 9 boys in Ms. Kade's second grade class. How many students are in the class?

4. There are 6 people in the Jones family. They decide to go out to eat with the Johnson family and make reservations at a restaurant for 11 people. How many people are in the Johnson family?

5. Thomas loves dill pickles. He ate 4 pickles at lunch and 5 at dinner. How many pickles did he eat?

6. If there are 7 days in one week, how many days are in two weeks?

7. Henry's parents are going on a 15-day business trip. Henry is going to stay with his grandpa and grandma part of the time and stay with his uncle and aunt part of the time. If he spends 7 days with his grandpa and grandma, how many days will he spend with his uncle and aunt?

8. There are 6 cars in a parking lot and 7 pickup trucks. How many vehicles is that?

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Day 4 – Doubles plus and minus 1 or 2

The strategy doubles plus or minus two is similar to the strategy doubles plus or minus one, but many children find it much more difficult. The following problems involve facts whose sum is one more or less than a double and facts whose sum is two more or less than a double. Verbally ask your child to determine the sum of the fact in column one and how he or she determined the answer. The second column describes two efficient reasoning strategies children might use to determine the sum of the fact by using doubles. It is not necessary for a child to describe both ways to think about each fact.

Double + or – 1 or 2	Thinking
5 + 6 = 11	• 5 + 5 is 10, so 5 + 6 is one more or 11 • 6 + 6 is 12, so 5 + 6 is one less or 11
5 + 7 = 9	• 5 + 5 is 10, so 5 + 7 is two more or 12 • 7 + 7 = 14, so 5 + 7 is two less or 12
5 + 4 = 9	• 4 + 4 is 8, so 5 + 4 is one more or 9 • 5 + 5 is 10, so 5 + 4 is one less or 9
3 + 4 = 7	• 3 + 3 is 6, so 3 + 4 is one more or 7 • 4 + 4 is 8, so 3 + 4 is one less or 7
5 + 3 = 8	• 3 + 3 is 6, so 5 + 3 is two more or 8 • 5 + 5 is 10, so 5 + 3 is two less or 8
9 + 8 = 17	• 8 + 8 is 16, so 9 + 8 is one more or 17 • 9 + 9 is 18, so 9 + 8 is one less or 17
7 + 6 = 13	• 6 + 6 is 12, so 7 + 6 is one more or 13 • 7 + 7 is 14, so 7 + 6 is one less or 13
6 + 8 = 14	• 6 + 6 is 12, so 6 + 8 is two more or 14 • 8 + 8 is 16, so 6 + 8 is two less or 16
8 + 7 = 15	• 7 + 7 is 14, so 8 + 7 is one more or 15 • 8 + 8 is 16, so 8 + 7 is one less or 15
7 + 9 = 16	• 7 + 7 is 14, so 7 + 9 is two more or 16 • 9 + 9 is 18, so 7 + 9 is two less or 16

Look for real-life examples of doubles and doubles plus and minus one or two as you go about your daily activities. Use the examples to create problems to discuss with your child.

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Day 5 – Doubles plus and minus 1 or 2

Each of the following word problems involves an addition or subtraction fact. Verbally ask your child to determine the answer to each problem and how he or she determined the answer. If your child is unsure of how to solve the problem, ask questions to help him or her think about the structure of the problem. The structure of addition and subtraction problems is different. If you know two or more parts, you can **add** to find the whole (how many in all). If you know one part and the whole (how many in all), you can **subtract** to find the other part. See Day 3 for sample questions to help your child think about the structure of the problems. Encourage your child to *Use Doubles* to determine the answer to each problem.

1. Spring has arrived and Mr. Lu's tulips are in full bloom. There are 5 red tulips and 6 yellow tulips in the front yard. How many tulips are in Mr. Lu's front yard?

2. There are 17 students in Mr. Webb's third grade class. Nine of the students are girls. How many are boys?

3. Truman checked out three library books and his sister checked out five. How many total library is that?

4. Meg walks 5 blocks to Molly's house and then another 7 blocks to the skate park. How far does she walk to the skate park?

5. Zaid sent 13 text messages one hour. Seven of the messages were to his friend Zan. How many messages were not sent to Zan?

6. There are 16 teams in the National League. Min has t-shirts from 9 of the teams. His goal is to collect a t-shirt from each of the National League baseball teams. How many more t-shirts must he buy to complete his collection?

7. Sam is collecting t-shirts from American League baseball teams. He bought 6 t-shirts last summer and 8 more t-shirts last fall. He now has exactly one t-shirt for each team in the American League. How many teams are in the American League?

8. Jose started a new running program. His goal is to run 25 miles each week. He ran 4 miles on Monday and 5 miles on Tuesday. How many total miles did he run so far this week?

9. Make up a word problem for $7 - 3 = 4$ and another word problem for $7 + 8 = 15$.

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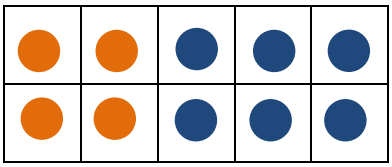
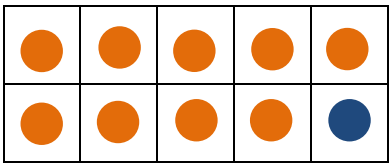
Day 6 – Use 10

Another strategy that helps students efficiently determine answers to difficult addition facts is called *Use 10*. In order to use this strategy, students must know the number combinations that add to 10 and be able to mentally add 10 and any single-digit number.

1. Ask your child what combinations of numbers add to 10. Print the ten frame shown on page 11 and encourage your child to represent the combinations with two different markers (two colors of paper, pennies and dimes, etc.) The arrangement of the markers is not important. Two examples are shown below.



4 + 6 = 10



9 + 1 = 10

Together represent each fact with the ten frame and make an organized list of the combinations. When finished, talk about the patterns you see in your list of facts. One way to organize the list and some patterns in the list are given below.

1 + 9 = 10
2 + 8 = 10
3 + 7 = 10
4 + 6 = 10
5 + 5 = 10
6 + 4 = 10
7 + 3 = 10
8 + 2 = 10
9 + 1 = 10

Patterns:

- If you reverse the order of the addends, the sum is still 10. (For example, 3 + 7 = 7 + 3) This is the commutative property of addition.
- As one addend increases by one the other addend decreases by one.
- If you subtract one addend from 10, you get the other addend. (For example, 10 – 9 = 1)

2. Verbally ask your child to determine the following sums. You may represent the problems by filling one ten frame and puting the extras outside the frame.

10 + 5 = 15	6 + 10 = 16	10 + 4 = 14
7 + 10 = 17	1 + 10 = 11	10 + 8 = 18
10 + 2 = 12	10 + 9 = 19	3 + 10 = 13

3. Ask your child how the problems he or she just solved for problem 2 are alike. Discuss the following patterns:
   - One of the addends is 10.
   - All of the problems are 10 plus a single digit number.
   - All of the answers have a 1 in the tens place.
   - The digit in the ones place of the answer is the number you added to 10.

   If your child is not able to add the numbers from problems 1 and 2 within a few seconds, review the problems periodically throughout the day. When the answers become automatic, your child is ready to learn the *Use 10* strategy.

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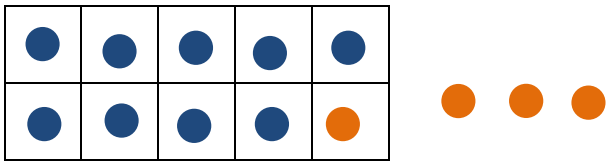
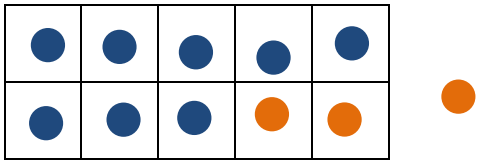
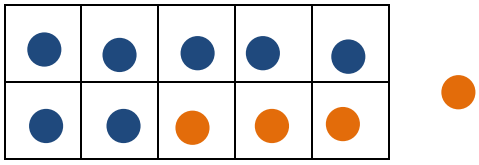
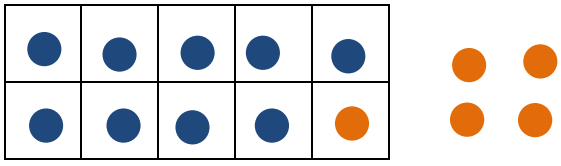
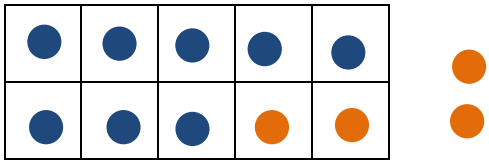
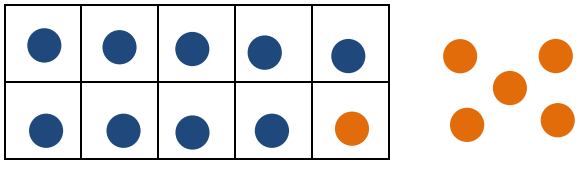
Day 7 – Use 10

Verbally ask your child to use the *Use 10* strategy to determine the answer to each of the addition facts in the table below. Encourage your child to use a ten frame and markers to explain his or her thinking. The first column gives the basic fact and the second column explains how a child might use a ten frame and markers to determine the answer.

Basic Addition Fact	Representation
9 + 4 = 13	 9 + 1 = 10 and 10 + 3 = 13
8 + 3 = 11	 8 + 2 = 10 and 10 + 1 = 11
4 + 7 = 11	 7 + 3 = 10 and 10 + 1 = 11
5 + 9 = 14	 9 + 1 = 10 and 10 + 4 = 14
4 + 8 = 12	 8 + 2 = 10 and 10 + 2 = 12
9 + 6 = 15	 9 + 1 = 10 and 10 + 5 = 15

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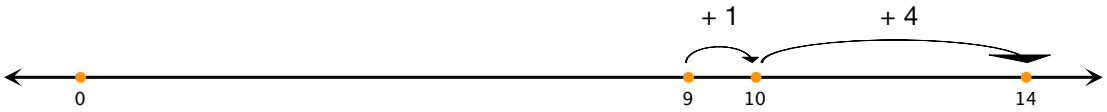
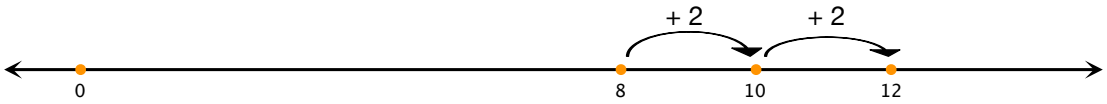
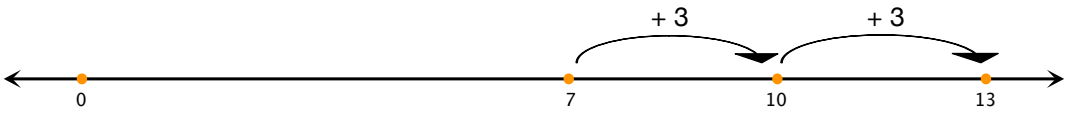
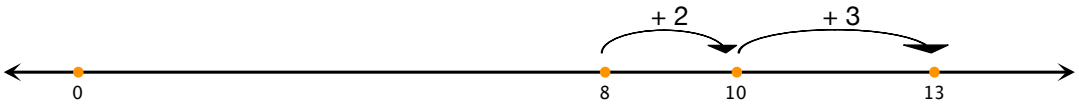
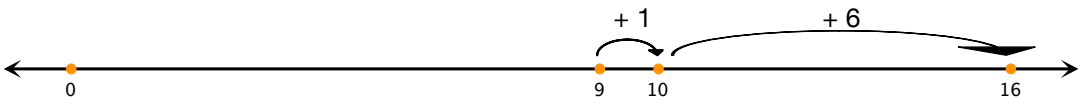
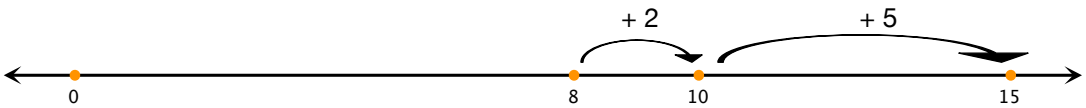
Day 8 – Use 10

A second way to represent the *Use 10* strategy is with an open number line. Verbally ask your child to use the *Use 10* strategy to determine the answer to each of the addition facts in the table below. Encourage your child to sketch an open number line to show his or her thinking. The first column gives the basic fact and the second column shows how a child might use an open number line to determine the answer.

Basic Addition Fact	Representation
9 + 5 = 14	 9 + 1 = 10 and 10 + 4 = 14
4 + 8 = 12	 8 + 2 = 10 and 10 + 2 =12
7 + 6 = 13	 7 + 3 = 10 and 10 + 3 = 13
8 + 5 = 13	 8 + 2 = 10 and 10 + 3 = 13
9 + 7 = 16	 9 + 1 = 10 and 10 + 6 = 16
8 + 7 = 15	 8 + 2 = 10 and 10 + 5 = 15

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Day 9 – Use Doubles or Use 10

The first column of the table below lists difficult addition facts. The second column describes the reasoning a child might use to efficiently determine the sum of the fact. For many of the facts, both the *Use Doubles* strategy and *Use Ten* strategy provide an efficient way to think about the problem. Verbally ask your child to determine the sum of each basic fact and explain how he or she determined the answer. It is not necessary for a child to describe more than one efficient way to think about each fact.

Basic Fact	Thinking
9 + 5 = 14	• 9 + 1 = 10 and 10 + 4 = 14
8 + 7 = 15	• 8 + 8 = 16, so 8 + 7 is one less or 15 • 7 + 7 = 14, so 8 + 7 is one more or 15 • 8 + 2 = 10 and 10 + 5 = 15
6 + 7 = 13	• 6 + 6 = 12, so 6 + 7 is one more or 13 • 7 + 7 = 14, so 6 + 7 is one less or 13 • 6 + 4 = 10 and 10 + 3 = 13 • 7 + 3 = 10 and 10 + 3 = 13
3 + 9 = 12	• 9 + 1 = 10 and 10 + 2 = 12
5 + 6 = 11	• 5 + 5 = 10 and 10 + 1 = 11 • 5 + 5 = 10, so 5 + 6 is one more or 11 • 6 + 6 = 12, so 5 + 6 is one less or 11
8 + 9 = 17	• 8 + 8 = 16, so 8 + 9 is one more or 17 • 9 + 9 =18, so 8 + 9 is one less or 17 • 9 + 1 = 10 and 10 + 7 = 17 • 8 + 2 = 10 and 10 + 7 = 17
9 + 4 = 13	• 9 + 1 = 10 and 10 + 3 = 13
5 + 7 = 12	• 5 + 5 = 10 and 10 + 2 = 12 • 7 + 3 = 10 and 10 + 2 = 12 • 5 + 5 = 10, so 5 + 7 is two more or 12 • 7 + 7 =14, so 5 + 7 is two less or 12
8 + 3 = 11	• 8 + 2 = 10 and 10 + 1 = 11
8 + 6 = 14	• 8 + 2 = 10 and 10 + 4 = 14 • 8 + 8 = 16, so 8 + 6 is two less or 14 • 6 + 6 = 12, so 8 + 6 is two more or 14
4 + 8 = 12	• 8 + 2 = 10 and 10 + 2 = 12
6 + 9 = 15	• 9 + 1 = 10 and 10 + 5 = 15
8 + 5 = 13	• 8 + 2 = 10 and 10 + 3 = 13
7 + 9 = 16	• 9 + 1 = 10 and 10 + 6 = 16 • 7 + 3 = 10 and 10 + 6 = 16

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Day 10 – Use Doubles or Use 10

Part 1: Beat the calculator: Print two copies of page 12, one for you and one for your child. Your child must determine the answer to each problem in row one mentally and write down the answer. You must determine the answer to each problem in row one with a calculator and write down the answer. Start at the same time. Whoever completes row one first and has all of the answers correct is the winner. Follow the same directions for row 2, only switch roles, you must use mental math and your child must use a calculator. Switch roles again for rows 3 and 4. Let your child determine whether to complete row 5 problems mentally or with a calculator.

Part 2: Word problems: Verbally ask your child to determine the answer to each word problem. Then ask how he or she determined the answer. If your child is unsure of how to solve the problem, ask questions to help him or her think about the structure of the problem. See Day 3 for sample questions. Encourage your child to *Use Doubles* or *Use Ten* to determine the answer to each problem.

1. Tessa and her mom played beat the calculator many times. Tessa won 8 times and Tessa's mom won 7 times. How many times did they play?

2. Jay took 40 seconds to correctly answer 9 addition problems and 4 subtraction problems. How many problems did he answer in 40 seconds?

3. Carlos and his dad played beat the calculator 17 times. When Carlos computed mentally he won the game, but when he used a calculator he lost. Carlos was the winner of 8 games. How many games did his dad win?

4. Mia beat the calculator 8 times and lost to the calculator 5 times. How many times did she compete against the calculator?

5. Sam played beat the calculator 10 times. He won more than he lost. How many games could he have won?

6. Zan played less than 20 games. He lost exactly the same number of games as he won. How many games might Zan have played?

7. Write an addition word problem for 8 + 4. Determine the answer.

8. Write a subtraction word problem for 9 – 3. Determine the answer.

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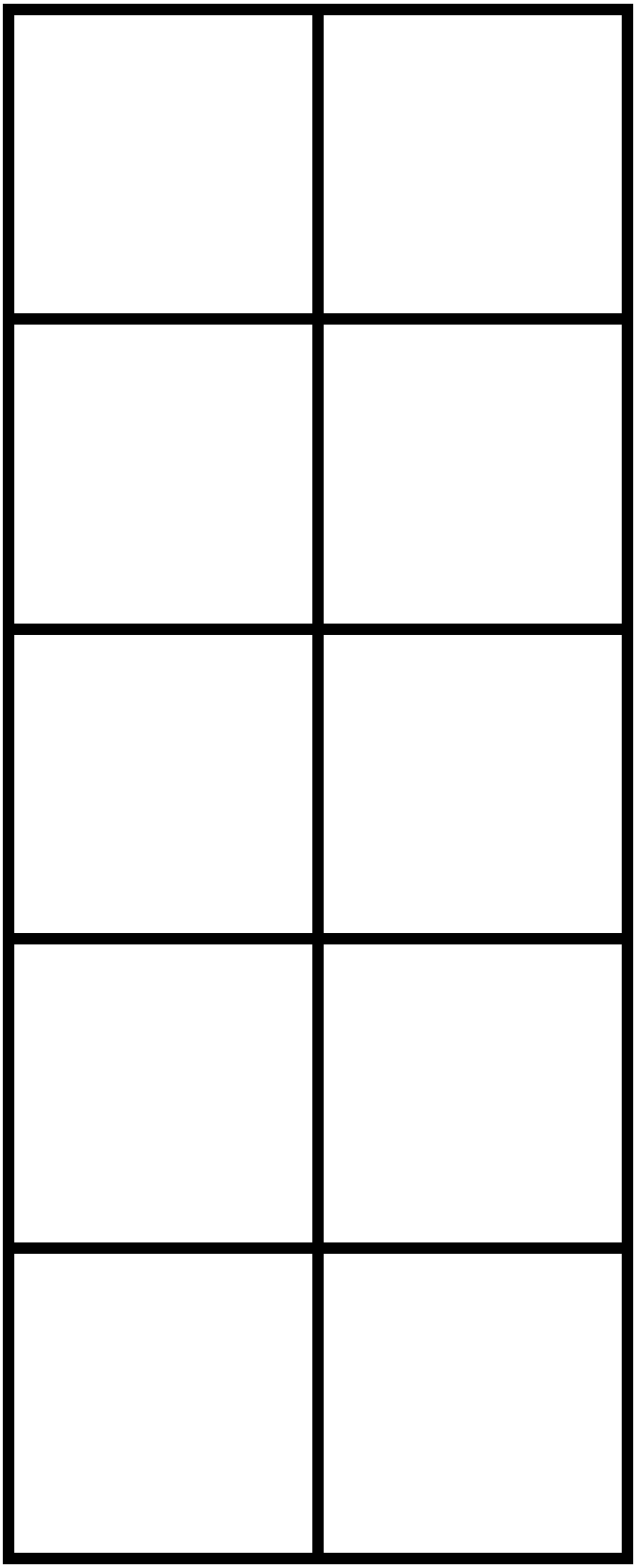
Ten Frame



Row 1: 8 + 3 = _____ 9 + 9 = _____ 16 - 8 =_____ 4 + 6 =_____ 5 + 6 =_____

Row 2: 4 + 8 = _____ 9 + 8 = _____ 18 - 9 =_____ 7 + 6 =_____ 8 + 8 =_____

Row 3: 6 + 8 = _____ 5 + 8 = _____ 10 - 5 =_____ 7 + 5 =_____ 7 + 8 =_____

Row 4: 7 + 9 = _____ 14 - 7 = _____ 9 + 5 =_____ 9 + 6 =_____ 7 + 7 =_____

Row 5: 9 + 3 = _____ 9 + 4 = _____ 4 + 7 =_____ 12 - 6 =_____ 3 + 5 =_____