Comparing Fractions Mentally

Two-week Learning Unit

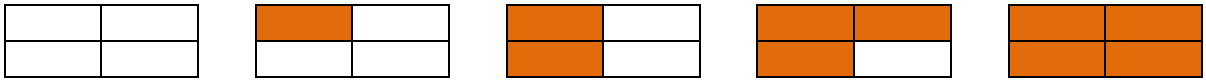
Directions for Parents and Tutors: The following mini-lessons focus on helping students develop thinking strategies to compare fractions mentally. Students who know a variety of strategies for comparing fractions can flexibly approach problems and solve problems accurately and efficiently. Learning a variety of thinking strategies also helps deepen students' understanding of fractions and helps students recognize when answers make sense.

Use the following 10 mini lessons over the course of two weeks. Spending 10 to 15 minutes each day over two weeks is much more effective than a few hour-long sessions.

Day 1 – Compare Fractions with Common Denominators

1. Have your child take five identical sheets of paper and fold each into four equal parts. Shade one fourth of the first sheet, two fourths of the second sheet, three fourths of the third sheet, and the entire fourth sheet. Do not shade the fifth sheet.

2. Ask your child to name the fraction represented by the shaded portion of each sheet of paper.



$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

3. Verbally ask your child to compare the following pairs of fractions and explain his or her answers. Encourage your child to use the shaded sheets of paper to make sense of the problems.

$\frac{3}{4} > \frac{1}{4}$ $\frac{0}{4} < \frac{3}{4}$ $\frac{2}{4} < \frac{3}{4}$ $\frac{3}{4} < \frac{4}{4}$ $\frac{4}{4} > \frac{1}{4}$

Possible explanation: The denominator represents the four equal parts of each sheet of paper. The numerator represents the number of shaded parts. Since the denominators are the same, you only need to compare the numerators. The more parts that are shaded the larger the fraction.

4. Verbally ask your child to compare the following pairs of fractions and explain his or her answers.

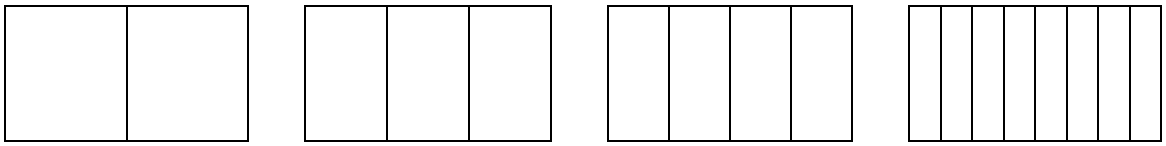
$\frac{2}{5} < \frac{5}{5}$ $\frac{5}{12} < \frac{7}{12}$ $\frac{2}{7} > \frac{0}{7}$ $\frac{7}{4} > \frac{5}{4}$ $\frac{12}{50} > \frac{11}{50}$

5. Ask your child to give a rule for comparing fractions with common denominators and explain why the rule will always work.

Possible explanation: If two fractions have a common denominator then both wholes are divided into the same number of parts and the size of the parts is equal. Thus the fraction representing more parts is the larger fraction.

Day 2 – Compare Fractions with Common Numerators

1. Take four identical sheets of paper and have your child fold them as shown below. Fold the first sheet in half, fold the second sheet in thirds, fold the third sheet in fourths, and fold the fourth sheet in eighths.



2. Ask your child what happens to the size of the parts as the denominator of a fraction gets larger and why this occurs. Encourage your child to use the folded sheets of paper to make sense of this question.

   Possible explanation: As the denominator gets larger, the parts get smaller. This is because the whole sheet of paper is being divided into more parts. It is like sharing a pizza equally among 4 people or 5 people. If the pizza is divided into 5 equal parts, the portions will be smaller than if divided into 4 equal parts.

3. Verbally ask your child to compare the following pairs of fractions mentally and explain his or her answers. Encourage your child to use the folded sheets of paper to make sense of the problems.

Fractions	Possible Explanation
$\frac{1}{2} > \frac{1}{4}$	Halves are larger than fourths, so one half is greater than one fourth.
$\frac{1}{3} < \frac{1}{2}$	Halves are larger than thirds, so one half is greater than one third.
$\frac{1}{8} < \frac{1}{3}$	Thirds are larger than eighths, so one third is greater than one eighth.
$\frac{1}{8} < \frac{1}{4}$	Fourths are larger than eighths, so one fourth is greater than one eighth.
$\frac{1}{3} > \frac{1}{4}$	Thirds are larger than fourths, so one third is greater than one fourth.

4. Verbally ask your child to compare the following pairs of fractions mentally and explain his or her answers. Encourage your child to use the folded sheets of paper to make sense of the problems.

Fractions	Possible Explanation
$\frac{2}{2} > \frac{2}{4}$	Halves are larger than fourths, so two halves is greater than two fourths.
$\frac{2}{3} > \frac{2}{8}$	Thirds are larger than eighths, so two thirds is greater than two eighths.
$\frac{3}{4} > \frac{3}{8}$	Fourths are larger than eighths, so three fourths is greater than three eighths.
$\frac{3}{3} > \frac{3}{4}$	Thirds are larger than fourths, so three thirds is greater than three fourths.
$\frac{2}{4} > \frac{2}{8}$	Fourths are larger than eighths, so two fourths is greater than two eighths.

Day 3 – Compare Fractions with Common Numerators or Denominators

Verbally ask your child to compare the following pairs of fractions mentally and explain his or her answers. Do not find common denominators. Think of dividing paper or pizza into equal-sized parts to help make sense of the problems.

Fractions	Possible Explanation
$\frac{2}{9} < \frac{2}{4}$	Fourths are larger than ninths, so 2 fourths is greater than 2 ninths.
$\frac{3}{7} > \frac{3}{8}$	Sevenths are larger than eighths, so 3 sevenths is greater than 3 eighths.
$\frac{6}{7} < \frac{5}{7}$	Six sevenths is greater than 5 sevenths.
$\frac{7}{12} > \frac{7}{23}$	Twelfths are larger than twenty-thirds, so 7 twelfths is greater than 7 twenty-thirds.
$\frac{7}{10} < \frac{7}{9}$	Ninths are larger than tenths, so 7 ninths is greater than 7 tenths.
$\frac{35}{50} < \frac{35}{43}$	Forty-thirds are larger than fiftieths, so 35 forty-thirds is greater than 35 fiftieths.
$\frac{12}{25} > \frac{9}{25}$	Twelve twenty-fifths is greater than 9 twenty-fifths.
$\frac{3}{4} > \frac{3}{100}$	Fourths are larger than thousandths, so 3 fourths is greater than 3 thousandths

Reconsider the last problem in the table, $\frac{3}{4} > \frac{3}{100}$. Ask your child to think about another way to justify the fact that three fourths is greater than 3 thousandths.

Possible explanation: Think about money. Three hundredths of a dollar is equal to three cents. Three fourths of a dollar is equal to 75 cents.

Day 4 – Compare Fractions to $\frac{1}{2}$

1. Verbally ask your child to name some fractions that are equal to one half and explain his or her reasoning.

   Possible explanation: Any fraction in which the numerator is one half the denominator is equal to one half. Examples include 2 fourths, 3 sixths, 12 twenty-fourths, etc.

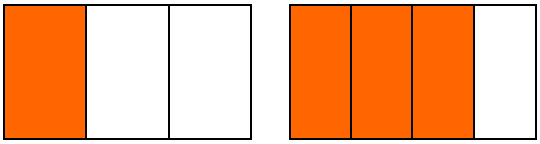
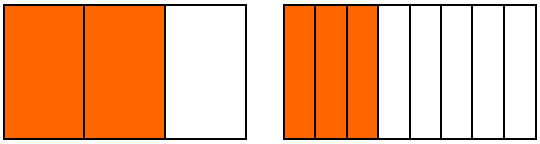
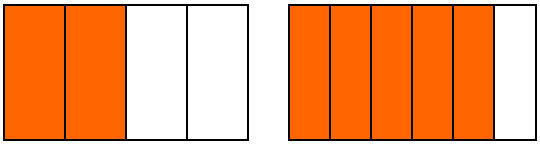
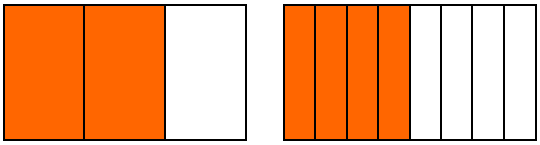
2. Verbally ask your child to name some fractions that are less than one half and explain his or her reasoning.

   Possible explanation: Any fraction in which the numerator is less than one half the denominator is less than one half. Examples include 1 fourth, 3 sevenths, 9 twenty-fourths, etc.

3. Verbally ask your child to name some fractions that are greater than one half and explain his or her reasoning.

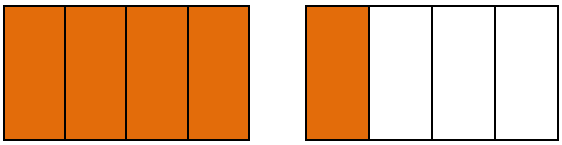
   Possible explanation: Any fraction in which the numerator is greater than one half the denominator is greater than one half. Examples include 3 fourths, 4 sevenths, 23 twenty-fourths, etc.

4. Verbally ask your child to compare the following fractions mentally without finding a common denominator and explain his or her reasoning. If your child is unsure about what to do, you may fold and shade paper to represent the pairs of fractions. Then compare each paper to one half.

Fractions	Possible Explanation	Paper Model
$\frac{1}{3} < \frac{3}{4}$	One third is less than one half, and three fourths is greater than one half.	
$\frac{2}{3} > \frac{3}{8}$	Two thirds is greater than one half and three eighths is less than one half.	
$\frac{2}{4} < \frac{5}{6}$	Two fourths is equal to one half and five sixths is greater than one half.	
$\frac{4}{8} < \frac{2}{3}$	Four eighths is equal to one half and two thirds is greater than one half.	

Day 5 - Compare Fractions to $\frac{1}{2}$ or 1

1. Verbally ask your child to name some fractions that are greater than 1 and explain his or her reasoning. If your child is unsure about what to do, fold and shade paper as shown below. Fold two sheets of paper into fourths and shade four fourths of one sheet and one fourth of the other. This represents one and one fourth or five fourths.



$$1\frac{1}{4} = \frac{5}{4}$$

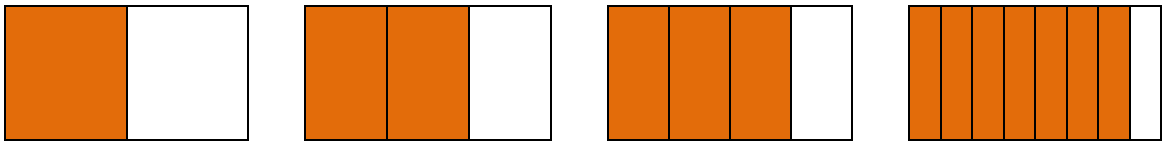
Possible explanation: Any fraction in which the numerator is greater than the denominator is greater than one. Examples include 5 fourths, 8 sevenths, 50 twenty-fourths, etc.

2. Verbally ask your child to compare the following pairs of fractions mentally without finding a common denominator and explain his or her reasoning. If your child is unsure use folded and shaded paper to represent the fractions.

Fractions	Possible Explanation
$\frac{7}{8} < \frac{6}{5}$	Seven eighths is less than one and six fifths is greater than one.
$\frac{7}{8} > \frac{2}{5}$	Seven eighths is greater than one half and two fifths is less than one half.
$\frac{3}{6} < \frac{2}{3}$	Three sixths is equal to one half and two thirds is greater than one half.
$\frac{13}{10} > \frac{3}{4}$	Thirteen tenths is greater than one and three fourths is less than one.
$\frac{7}{8} < \frac{3}{2}$	Seven eighths is less than one and three halves is greater than one.
$\frac{5}{3} > \frac{3}{5}$	Five thirds is greater than one and three fifths is less than one.
$\frac{4}{7} > \frac{3}{5}$	Four sevenths is greater than one half and three fifths is less than one half.
$\frac{5}{4} > \frac{9}{8}$	Both fractions are greater than one. Five fourths is equal to one and one fourth. Nine eighths is equal to one and one eighth. One fourth is greater than one eighth.

Day 6 – Compare Fractions to 1

1. Take four identical sheets of paper and have your child fold them as shown below. Fold the first sheet in half, fold the second sheet in thirds, fold the third sheet in fourths, and fold the fourth sheet in eighths. Shade all but one part of each sheet.



$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{7}{8}$

2. Verbally ask your child to compare the following pairs of fractions mentally without finding a common denominator and explain his or her answers. Encourage your child to use the folded sheets of paper to make sense of the first three problems and extend the same thinking to the last three problems.

Fractions	Possible Explanation
$\frac{3}{4} > \frac{2}{3}$	Both fractions are one part less than one. One fourth is less than one third. Three fourths is missing a smaller part.
$\frac{7}{8} > \frac{1}{2}$	Both fractions are one part less than one. One eighth is less than one half. Seven eighths is missing a smaller part.
$\frac{7}{8} > \frac{2}{3}$	Both fractions are one part less than one. One eighth is less than one third. Seven eighths is missing a smaller part.
$\frac{3}{4} < \frac{9}{10}$	Both fractions are one part less than one. One tenth is less than one fourth. Nine tenths is missing a smaller part.
$\frac{99}{100} > \frac{9}{10}$	Both fractions are one part less than one. One hundredth is less than one tenth. Ninety-nine hundredths is missing a smaller part.
$\frac{6}{8} < \frac{8}{10}$	Both fractions are two parts less than one. Two tenths is less than two eighths. Eight tenths is missing smaller parts.

Day 7 – Compare More than Two Fractions

Part 1: Have your child write down the following five fractions and then place them in order from smallest to largest. Encourage your child to compare the fractions mentally without finding a common denominator. While your child is working on this task, complete the task yourself. Have your child share the thinking strategies he or she used to order the fractions. Discuss the similarities and differences between the strategies you and your child use.

$\frac{3}{4}$ $\frac{1}{2}$ $\frac{9}{8}$ $\frac{2}{7}$ $\frac{3}{5}$

Possible explanation:

- $\frac{9}{8}$ is the only fraction greater than one, so it is the largest.
- $\frac{2}{7}$ is the only fraction less than one half, so it is the smallest.
- $\frac{1}{2}$ is the next fraction since all of the remaining fractions are greater than one half.
- $\frac{3}{5}$ and $\frac{3}{4}$ are greater than one half but less than one. Fourths are larger than fifths, so three fourths is greater than three fifths.

Correct order (smallest to largest): $\frac{2}{7}$ $\frac{1}{2}$ $\frac{3}{5}$ $\frac{3}{4}$ $\frac{9}{8}$

Part 2: Now have your child write down and order the following fractions mentally, without finding a common denominator. Again complete the task yourself and when finished, discuss possible strategies with your child.

$\frac{3}{10}$ $\frac{1}{2}$ $\frac{99}{100}$ $\frac{3}{2}$ $\frac{3}{7}$ $\frac{16}{15}$ $\frac{4}{5}$

Possible explanation:

- $\frac{16}{15}$ and $\frac{3}{2}$ are both one part more than one. Halves are larger than fifteenths so three halves is the largest.
- $\frac{99}{100}$ and $\frac{4}{5}$ are both greater than one half and one part from one. Hundredths are smaller than fifths, so 99 hundredths is missing a smaller part and closer to one.
- $\frac{3}{7}$ and $\frac{3}{10}$ are both less than one half. Tenths are smaller than sevenths so three tenths is less than three sevenths.

Correct order (smallest to largest): $\frac{3}{10}$ $\frac{3}{7}$ $\frac{1}{2}$ $\frac{4}{5}$ $\frac{99}{100}$ $\frac{16}{15}$ $\frac{3}{2}$

Day 8 – Compare Fractions
Verbally ask your child to mentally compare the following pairs of fractions and explain how he or she determined the answer.

Fractions	Reasoning
$\frac{9}{8} > \frac{13}{12}$	Strategy: Compare to one Reasoning: Nine eighths is one eighth greater than one. Thirteen twelfths is one twelfth greater than one. One eighth is greater than one twelfth.
$\frac{12}{13} < \frac{14}{15}$	Strategy: Compare to one Reasoning: Both fractions are one part less than one. One fifteenth is less than one thirteenth. Fourteen fifteenths is missing a smaller part.
$\frac{7}{12} > \frac{7}{15}$	Strategy: Common numerators One twelfth is greater than one fifteenth. Seven larger parts is greater than seven smaller parts.
$\frac{57}{100} > \frac{3}{8}$	Strategy: Compare to one half Reasoning: Three eighths is less than one half and fifty-seven hundredths is more than one half.
$\frac{7}{16} < \frac{9}{12}$	Strategy: Compare to one half Reasoning: Seven sixteenths is less than one half, and nine twelfths is more than one half. Another approach: Nine twelfths has larger parts and more parts than seven sixteenths.

Extra Challenge: Verbally ask your child to mentally compare the following pairs of fractions and explain how he or she determined the answer.

Fractions	Reasoning
$\frac{3}{5} < \frac{13}{15}$	Strategy: Compare to one Reasoning: Both fractions are two parts less than one. One fifth is greater than one fifteenth, so two fifths is greater than two fifteenths. Thirteen fifteenths is missing two smaller parts.
$\frac{9}{16} < \frac{7}{8}$	Strategy: Compare to three fourths Reasoning: Twelve sixteenths equals three fourths, so nine sixteenths is less than three fourths. Six eighths equals three fourths, so seven eighths is greater than three fourths.

Day 9 – Problems in Context

Each of the following word problems involves comparing fractional amounts. Determine the answer to each question by comparing the fractions mentally without finding a common denominator. If the problem does not provide you with enough information to answer the question, describe what information is missing.

1. Lu decided to buy his father a new baseball cap. He found one he liked and bought it without considering the size. When he got home he saw it was a size $7\frac{3}{4}$. His dad wears a size $7\frac{7}{8}$. Was the cap too small or too large for his father?

   Answer and explanation: Both sizes are one part less than eight. One eighth is less than one fourth. Seven and seven eighths is missing a smaller part, so it is closer to 8. The hat will be too small for Lu's father.

2. Jack and his sister attended two different pizza parties. When they got home they argued about who ate more pizza. Jack said he ate one half of a pizza and Misha said she ate five sixths of a pizza. Who ate more pizza?

   Answer and explanation: It is not possible to determine the answer because we do not know the size of the pizzas. We might be comparing a large 16-inch pizza to a small individual 6-inch pizza.

3. Misha and her friends all shared one large pizza. Misha ate one eighth of the pizza, Tessa ate one fourth of the pizza, and Liza ate one half of the pizza. Who ate the most? Who ate the least?

   Answer and explanation: Since all of the girls were talking about the same pizza, we can compare the fractional amounts. Halves are larger than fourths or eighths, so one half is larger than one fourth or one eighth. Fourths are larger than eighths, so one fourth is larger than one eighth. Liza ate the most pizza and Misha ate the smallest amount of pizza.

4. Umar made himself an omelet for breakfast. He used one fourth of a full carton of eggs. His brother woke up a little later and also made himself an omelet for breakfast. He used one sixth of a full carton of eggs. Both Umar and his brother used the same number of eggs. How is this possible?

   Answer and explanation: If Umar used eggs from a carton that holds 12 eggs and Umar's bother used eggs from a carton that holds 18 eggs, both would have used 3 eggs in their omelets.

Day 10 – Fraction Race

Print two copies of this page, one for you and one for your child. Your child must determine the answer to each problem in row one mentally. You must determine the answer to each problem in row one with paper and pencil by finding a common denominator. Start at the same time. Whoever completes row one first and has all of the answers correct is the winner. Follow the same directions for row 2, only switch roles, you must use mental math and your child must find a common denominator. Switch roles again for rows 3 and 4. Let your child determine whether to complete row 5 problems mentally or by finding a common denominator.

Row 1	$\frac{3}{4}$ $\frac{3}{14}$	$\frac{4}{9}$ $\frac{7}{10}$	$\frac{7}{8}$ $\frac{20}{21}$
Row 2	$\frac{14}{15}$ $\frac{11}{12}$	$\frac{7}{13}$ $\frac{7}{10}$	$\frac{2}{5}$ $\frac{7}{12}$
Row 3	$\frac{6}{5}$ $\frac{26}{25}$	$\frac{9}{20}$ $\frac{2}{3}$	$\frac{3}{4}$ $\frac{3}{7}$
Row 4	$\frac{5}{17}$ $\frac{5}{12}$	$\frac{5}{6}$ $\frac{3}{8}$	$\frac{9}{8}$ $\frac{10}{9}$
Row 5	$\frac{11}{24}$ $\frac{3}{5}$	$\frac{1}{4}$ $\frac{1}{11}$	$\frac{7}{8}$ $\frac{8}{9}$