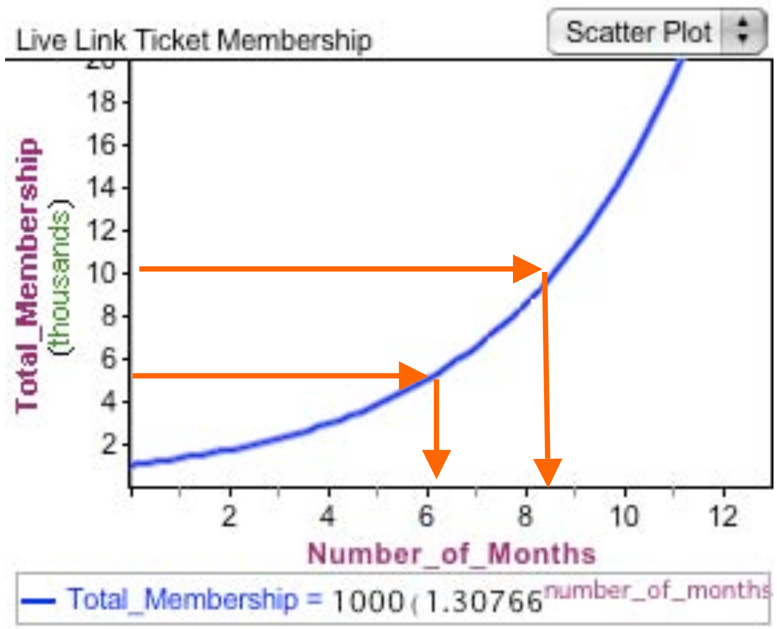
Solving for Exponents

Use What You've Learned
Answers and Explanations

After watching the video, *Solving for Exponents,* complete the following problems.

Note: Some of these problems will require the use of a calculator with a "log" function on it.

1. Assume that the total membership for Live Link Tickets was given by this graph:



   a. In what month does Live Link Tickets have 5000 members?
   Estimate from the graph: Month 6

   b. Estimate when Live Link Tickets has 10000 members?
   Estimate from the graph: between Months 8 and 9

2. My bank just sent me an offer for a savings account with an interest rate of 3 percent. That seems low to me, but the banker is claiming it is quite generous. If I start by investing \$100, after n years I will have D dollars, where $D = 100(1.03)^n$.
   a. How much money will I have after 3 years?
   $D = 100(1.03)^3$
   $D = \$109.27$

b. How many years will it take for me to have $125?

$125 = 100(1.03)^n$

$\frac{125}{100} = \frac{100(1.03)^n}{100}$

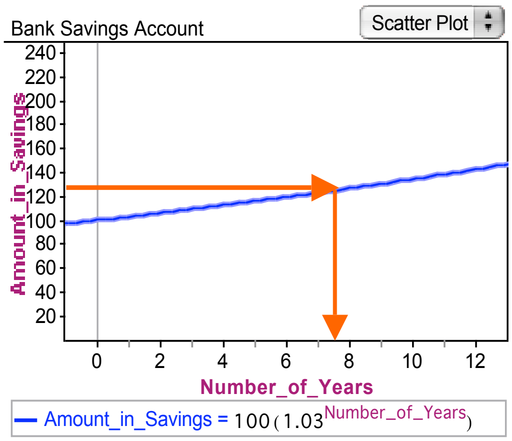
$1.25 = 1.03^n$

$\log 1.25 = \log(1.03)^n$

$\log 1.25 = n \cdot \log(1.03)$

$\frac{\log 1.25}{\log 1.03} = n$

$7.55 \text{ years} \approx n$



c. How long will it take for my money to double?

$200 = 100(1.03)^n$

$\frac{200}{100} = \frac{100(1.03)^n}{100}$

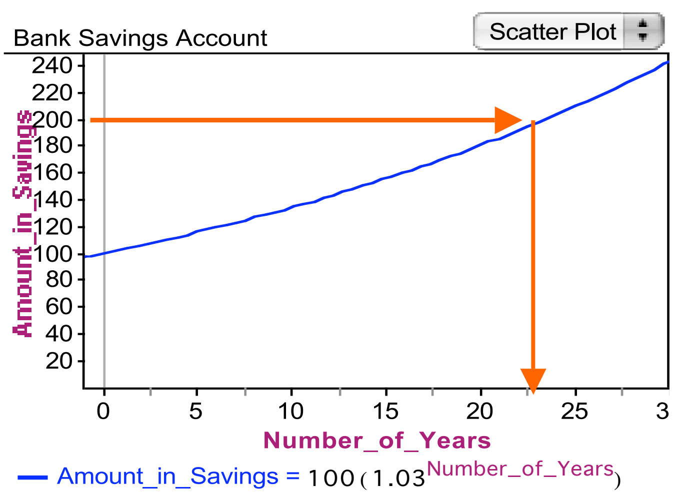
$2 = 1.03^n$

$\log 2 = \log(1.03^n)$

$\log 2 = n \cdot \log 1.03$

$\frac{\log 2}{\log 1.03} = n$

$23.4 \text{ years} \approx n$



3. Take a piece of paper and fold it in half. You now have a sheet of paper twice as thick as the one with which you started. Keep folding it over and over again. Assuming that the paper is 0.002 inches thick, the thickness of the folded sheet is given by $T = 0.002(2^n)$ where n is the number of folds and T is the thickness in inches.

a. How thick would the paper be after 5 folds?

$T = 0.002(2^n)$

$T = 0.002(2^5)$

$T = 0.064 \text{ inches}$

b. How many folds would it take to make the paper over a mile thick?

One mile is 63,360 inches.

$$63,360 = 0.002(2^n)$$
$$\frac{63,360}{0.002} = 2^n$$
$$31,680,000 = 2^n$$
$$\log 31,680,000 = \log(2^n)$$
$$\log 31,680,000 = n \cdot \log 2$$
$$\frac{\log 31,680,000}{\log 2} = n$$
$$25 \text{ folds} \approx n$$

c. How many folds can you do on a real sheet of paper? Go ahead and do the experiment. Does it matter how large of a piece of paper you start with?

It does not matter how large of a piece of paper you start with. Most people will not be able to do more than 7 folds.

4. One student offered a slightly different way to solve for exponents. State a justification or property for each step.

a. $4^x = 20$

$(10^{\log 4})^x = 10^{\log 20}$	rewrite 4 and 20 with a base of 10
$10^{(\log 4)x} = 10^{\log 20}$	property of exponents
$(\log 4)\,x = \log 20$	exponents are equal
$x = \dfrac{\log 20}{\log 4}$	divide both sides by equal amount
$x \approx 2.16$	

b. $3^x = 90$

$(10^{\log 3})^x = 10^{\log 90}$	rewrite 3 and 90 with a base of 10
$10^{(\log 3)x} = 10^{\log 90}$	property of exponents
$(\log 3)\,x = \log 90$	exponents are equal
$x = \dfrac{\log 90}{\log 3}$	divide both sides by equal amount
$x \approx 4.10$	

c. This student decided a short cut might be to take the "log of both sides." Why does this work?

When rewriting a number using a base of ten, the common logarithm is used since these are inverse relationships. So, there will always be the log of both values from the original problem.