Introduction to Logarithms
Use What You've Learned
Answers and Explanations

1. The definition of logarithm is if $a^x = y$, then $\log_a y = x$, and if $\log_a y = x$, then $a^x = y$.

   a. Complete the tables for an exponential function base 10 and a logarithmic function base 10.

x	10^x
0	$10^0 = 1$
1	10^1
2	$10^2 = 100$
3	$10^3 = 1000$
4	$10^4 = 10000$
5	$10^5 = 100000$
6	$10^6 = 1000000$
7	$10^7 = 10000000$
8	$10^8 = 100000000$
9	$10^9 = 1000000000$
10	$10^{10} = 10000000000$

y	$\log_{10} y$
1	0
10	1
100	2
1000	3
10000	4
10^5	5
10^6	6
10^7	7
10^8	8
10^9	9
10^{10}	10

   b. Ten raised to what power is 1,000,000?
   Ten raised to the sixth power is 1,000,000.

   c. How can the definition of logarithms help you find $\log_{10} 1000000$?
   Log_{10} 1,000,000 is asking the question, "Ten raised to what power is 1,000,000?"

   d. Using the table, estimate $\log_{10} 99{,}932$ to the nearest whole number.
   Log_{10} 99,932 is asking the question, "Ten raised to what power is 99,932?" An exponent that is a little less than 5 is the solution since 10^4 is 10,000 and 10^5 is 100,000. The best whole number estimate would be 5.

   e. Using the table, estimate $10^{3.1}$.
   10^3 is 1000 and 10^4 is 10000. $10^{3.1}$ is closer to 10^3 than 10^4, so 1200 is a good estimate.

2. Complete the tables below. The base is **three** in both tables.

x	3^x
0	1
1	3
2	9
3	27
4	81
5	243

y	$\log_3 y$
1	0
3	1
9	2
27	3
81	4
243	5

a. Without using a calculator, compute the following base **three** logarithms.

i) $\log_3(81) = 4$

$3^4 = 81$

ii) $\log_3(243) = 5$

$3^5 = 243$

iii) $\log_3(1) = 0$

$3^0 = 1$

iv) $\log_3\left(\frac{1}{3}\right) = -1$

$3^{-1} = \frac{1}{3}$

v) $\log_3\left(\frac{1}{9}\right) = -2$

$3^{-2} = \frac{1}{9}$

3. Moore's Law states, informally, that the computing power of a chip doubles every two years.

a. Make a table showing how the computing power of a chip increases, where *n* is the number of doubling periods.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64

b. According to Moore's Law, how long will it take the computing power of a chip to increase by a factor of 64?
This question can be rewritten, what is the value of n if $\log_2 64 = n$ (where n is the number of doubling periods). Since $2^6 = 64$, n = 6. So, there will be 6 two-year periods which totals 12 years.

c. According to Moore's Law, by what factor will the computing power of the chip increase in 16 years?
This question can be rewritten, what is the value of f if $\log_2 f = 8$ (where f is the factor of increase of the computing chip)? Note that 8 is the number of two-year periods for 16 years. The computing power of the chip is increased by a factor of 256 ($f = 2^8$).

4. Assume the population (*p*) of a virus in a human body triples every hour.

a. If we start with 1 virus in a body, how many will there be in three hours?

t	3^t
0	1
1	3
2	9
3	27
4	81
5	243
6	729

Use the equation $\log_3(p) = t$, where t is the number of hours and p is the population of the virus. For this problem we need to find $\log_3 p = 3$. Rewriting, $3^3 = p$, or $27 = p$. So, the population of the virus is 27 in three hours.

b. How long will it take for the population of viruses to be 243?
In the equation $\log_3 243 = t$, $t = 5$ hours. It may be helpful to think of $3^t = 243$. It will take 5 hours for the population of viruses to be 243.

c. How many viruses will there be in one day?
This can be represented as $\log_3(p) = 24$. Note that 24 hours is used instead of one day since t = hours. Rewriting the equation using the definition of logarithms $3^{24} = p$, or p = 282,429,536,481 viruses.

d. Is the equation below a valid representation for the number of viruses in a human body? Why or why not?
$t = \log_3(p)$ (*t* = time in hours, *p* = population)
Yes, since $p = 3^t$, we have $\log_3(p) = t$.

5. The following is a graph of $y = 4^x$. Use the graph to estimate $\log_4(8000)$.
   Express $\log_4(8000)=x$ in exponential form by using the definition of logarithms. $4^x=8000$. Then, use the graph to find an estimate. Start at y=8000 and move horizontally to the graph of the function. Then, move directly down to the x-axis. The value of x is estimated to be a little less than 6.5.