MAKING SENSE OF MATHEMATICS

After watching the video, Solving for Exponents, complete the following problems.

1. Estimate a solution for $5^{x}=3^{2 x+1}$ using a graph. Then, solve using symbolic reasoning. Solution:

Graph both functions using an appropriate window.


Using symbolic reasoning:
$5^{x}=3^{2 x+1}$
$\log \left(5^{x}\right)=\log \left(3^{2 x+1}\right)$
$x \log 5=(2 x+1) \log 3$
$(\log 5) x-(2 \log 3) x=\log 3$
$(\log 5-2 \log 3) x=\log 3$
$x=\frac{\log 3}{\log 5-2 \log 3} \approx-1.869$

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2. According to data from the U.S. Department of Agriculture, beef consumption in the United States (in billions of pounds) x years after 1900 can be approximated by $-154.41+90.68 \log x$.
a) According to this model, how much beef was consumed in 2000?
beef consumption $=-154.41+90.68(\log 100)$
beef consumption $=-154.41+90.68 \cdot 2$
beef consumption $=26.95$ billion pounds
b) According to this model, when will beef consumption reach 50 billion pounds per year?

```
50=-154.41+90.68(log}x
204.41 = 90.68(知 x)
204.41
    0.68
    204.41
10 90.68}=x\mathrm{ by definition of logarithms
179.55 \approxx
```

This means in approximately 179.55 years from the year 1900 or about the year 2079 beef consumption will reach 50 billion pounds per year.
3. The following is a graph of $x$ versus $\log _{b} x$. What is the value of $b$ ? How do you know?


There are many ways to figure out the value of b . One way is to look at individual data points such as $(9,2)$ and substitute them into the equation.

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$$
\begin{aligned}
& y=\log _{b} x \\
& 2=\log _{b} 9 \\
& b^{2}=9 \\
& b=3
\end{aligned}
$$

4. Find the value of $b$ given the following table of values. You don't need a calculator to find it!

| $\mathbf{x}$ | $\log _{\mathrm{b}} \mathbf{x}$ |
| :---: | :---: |
| 1 | 0 |
| 18 | 1.6131 |
| 36 | 2.0000 |
| 50 | 2.1833 |

There are many ways to figure out the value of b . One way is to look at individual data points such as $(36,2)$ and substitute them into the equation.

$$
\begin{aligned}
& y=\log _{b} x \\
& 2=\log _{b} 36 \\
& b^{2}=36 \\
& b=6
\end{aligned}
$$

5. Solve the following equation: $10^{x^{2}-3 x+6}=10,000$

It may be helpful to graph both sides of the equation and look at the graphical solution.
A symbolic solution is provided below.

$$
\begin{aligned}
& 10^{x^{2}-3 x+6}=10,000 \\
& \log \left(10^{x^{2}-3 x+6}\right)=\log (10,000) \\
& x^{2}-3 x+6=4 \\
& x^{2}-3 x+2=0 \\
& (x-2)(x-1)=0 \\
& x=2, x=1
\end{aligned}
$$

The solution can also be found by rewriting 10,000 with a base of 10 to $10^{4}$. Since the exponents must be equal to each other,

$$
\text { then } \begin{aligned}
& x^{2}-3 x+6=4 \\
& x^{2}-3 x+2=0 \\
& (x-2)(x-1)=0 \\
& x=2, x=1
\end{aligned}
$$

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6. Solve the following equation: $10^{3 x+2}=10^{5}$

This equation can be solved by inspection. Since both sides of the equation are base ten values, the exponents are equal to each other. This means $3 x+2=5$, so $x=1$. If this is not noticed, then take the log of both sides of the equation; this is the result:

$$
\begin{aligned}
& 10^{3 x+2}=10^{5} \\
& \log \left(10^{3 x+2}\right)=\log \left(10^{5}\right) \\
& 3 x+2=5 \\
& x=1
\end{aligned}
$$

