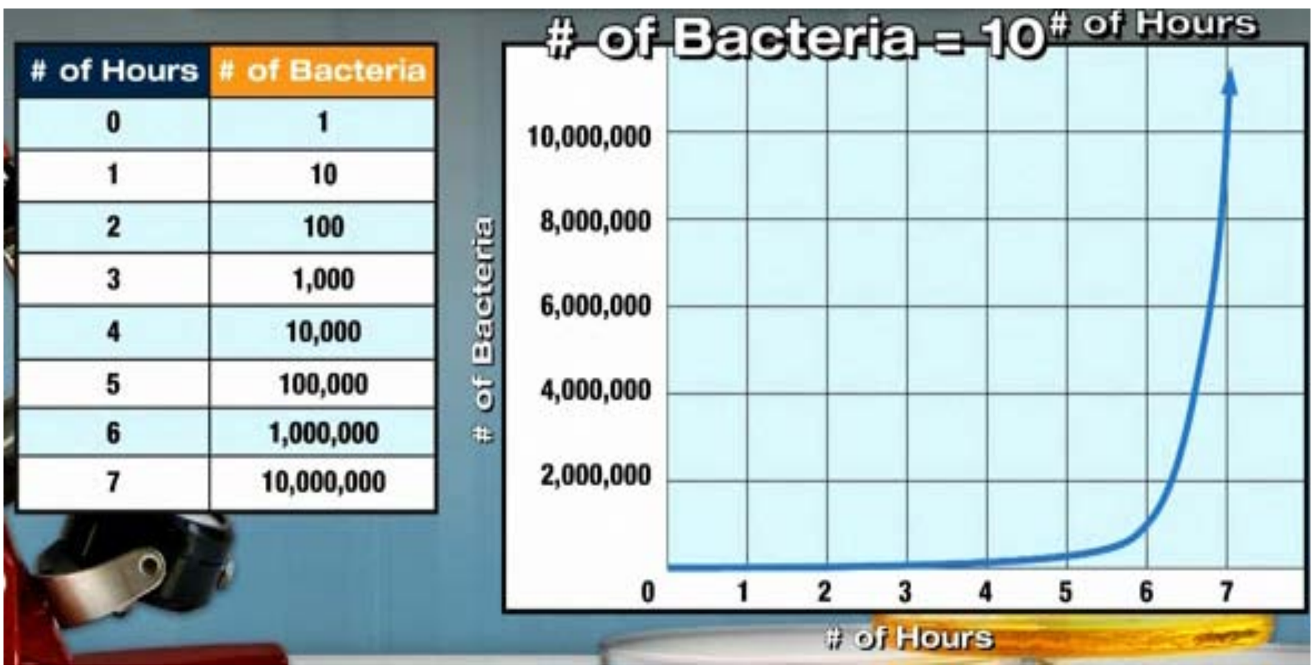
Introduction to Logarithms

A Closer Look at the Video

After watching the *Introduction to Logarithms* video, make sense of the mathematics by reading through the problem situation and solution. Use the comments and questions in bold to help you create tables, graphs, and equations for the situation.

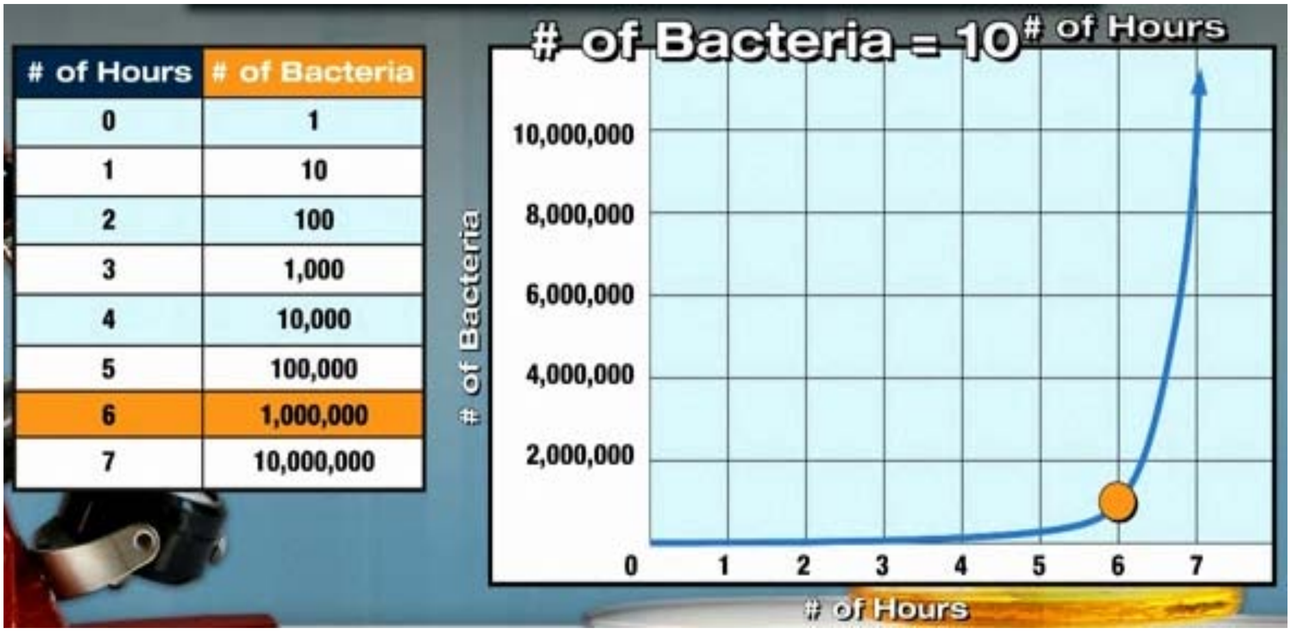
Problem: Dr. Parker, a biologist, wants to determine the number of hours it will take a bacteria sample with one cell to grow to five million cells if the bacteria is growing exponentially by a factor of ten per hour.

How can we create a table and graph representing the growth of the bacteria?
We'll call the time we begin growing our sample, hour zero, and start with a single bacterium. After the first hour, we would have ten bacteria, and after the second, we would have 100 bacteria. Continuing the exponential pattern, number of bacteria = $10^{\text{number of hours}}$, we should get a table and graph that look like this.



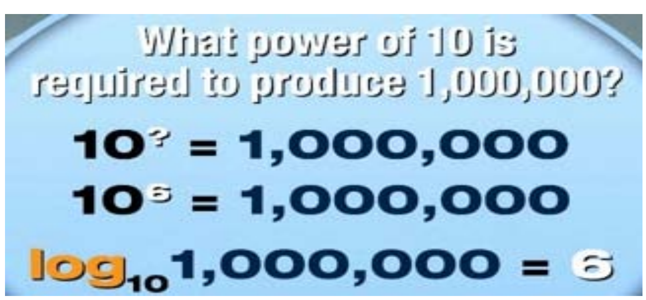
# of Hours	# of Bacteria
0	1
1	10
2	100
3	1,000
4	10,000
5	100,000
6	1,000,000
7	10,000,000

How long until the sample reaches one million cells?
By locating one million bacteria in the table or graph, we can see that if this exponential pattern continues, there will be one million bacteria after six hours.

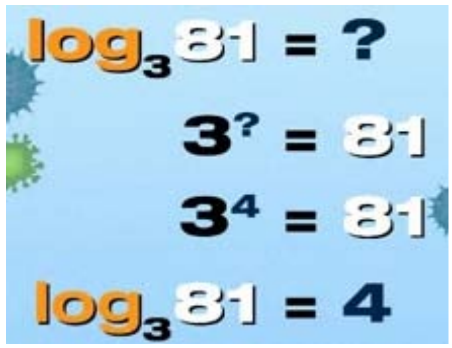


Why can't we use the table to determine the time it will take for the bacteria to grow to five million cells?
We don't see five million bacteria in the table; the table goes from one million to ten million. To find five million, we will need to use logarithms.

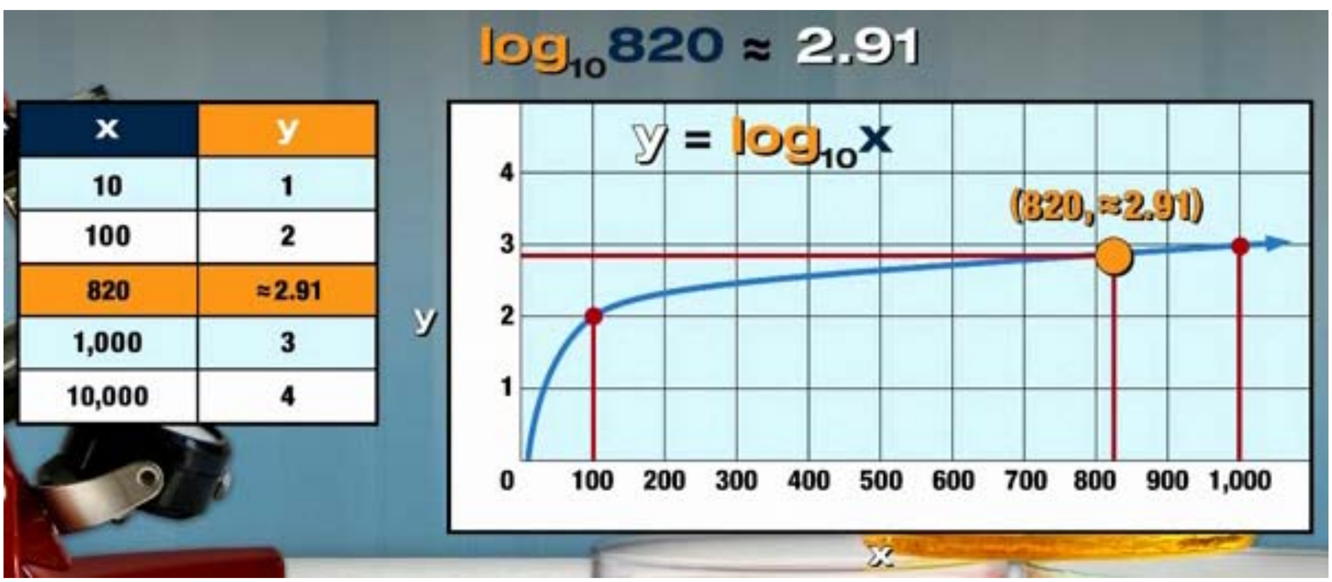
What question do logarithms answer?
Let's use the answer to the question: "How long until the sample reaches one million cells?" to help us understand this problem. To answer this question, we need to know what power of ten is required to produce one million. In this case, the exponent is six. Without knowing it, we just used a logarithm. The logarithm gives us the exponent necessary to produce a desired result. **Logarithms answer the question: "What power of the base is required to produce a given value?"** In our example, the logarithm of one million base ten is six, and the notation looks like this: $\log_{10} 1{,}000{,}000 = 6$.



If we were asked to find the $\log_3 81$ (log base 3 of 81), what question would we ask and what would the answer be?
We would ask the question: Three raised to what power is 81? To find the answer, we take 3 x 3 x 3 x 3 is 81. So, 3 raised to the fourth power equals 81 and the answer to our question is the exponent, 4. That means 4 is the logarithm of 81 when the base 3.

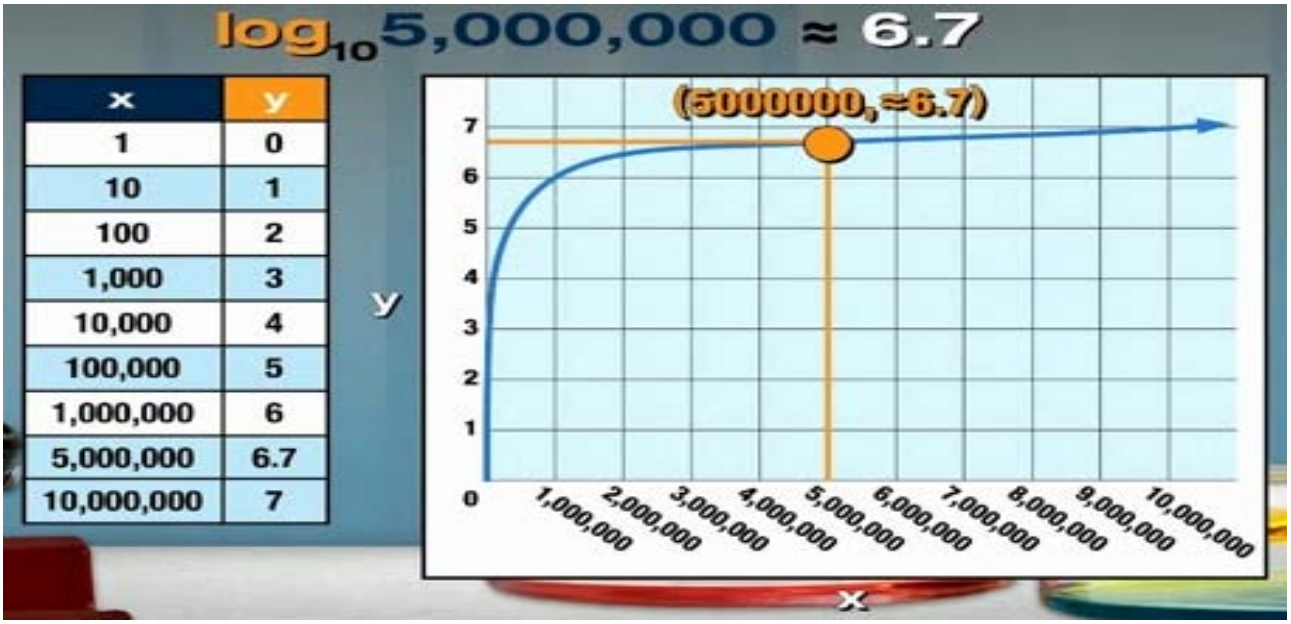


Try estimating $\log_{10}820$.
First, ask yourself, 10 to what power is 820? It's not an easy answer, but you do know that $10^2 = 100$, and $10^3 = 1000$. So, $\log_{10}820$ has to be between 2, which is too small and 3, which is too big. The actual value is close to 2.91.



Now, let's go back to the original question about when the bacteria will reach five million cells. How can we estimate the answer to this question?
Looking at the number of hours and bacteria in the table and graph below, we see they are plotted as a logarithm. We can estimate the answer knowing that 10^6 is one million - that's too

small, and 10^7 is ten million - that's too big. So, we know that $\log_{10}5{,}000{,}000$ is between six and seven. It turns out that in approximately 6.7 hours, there will be five million bacteria.



How can we generalize our logarithmic equations?

To help us generalize, let's look at another example. We know if $10^{6.7} \approx 5{,}000{,}000$, then $\log_{10}5{,}000{,}000 \approx 6.7$. Using variables, if a base, *a,* raised to an exponent, *y,* equals a given number, *x*, then $\log_a x = y$.